MULTICHANNEL UNITARITY AND THE EFFECTIVE CROSS SECTION OF $p + {}^{11}B \rightarrow 3\alpha$: IMPLICATIONS FOR THE LAWSON CRITERION

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The reaction $p + {}^{11}B \rightarrow {}^{12}C^* \rightarrow 3\alpha$ ($Q = +8.682$ MeV) is the leading candidate for aneutronic thermonuclear fusion without primary neutrons. A precise effective cross section $\sigma_{3\alpha}(E)$ is a prerequisite for an honest assessment of the Lawson criterion for $p + {}^{11}B$ plasma devices, including laser-driven experiments by HB11 Energy [1] and magnetic confinement (LHD) [2].

We demonstrate that single-channel and geometrically motivated descriptions of $\sigma_{3\alpha}$ are fundamentally incompatible with S-matrix unitarity $S^\dagger S = 1$, which constrains the total probability flux across all open channels simultaneously. At $E_p \approx 2.65$ MeV ($J^\pi = 2^+$, $E_x \approx 18.39$ MeV in ${}^{12}C^*$), at least 6 channels are open. The 3α branching ratio $BR_{3\alpha}(E_r) \approx 40$–43 % [3], not 100 % as in the naive geometric estimate $\sigma_{geom} = \pi\bar{\lambda}^2 g_J$ ($BR_{3\alpha} \equiv 1$), leading to a systematic overestimate by a factor $1/BR_{3\alpha} \approx 2.3$.

A systematic progressive R-matrix analysis is presented (Fig. 1): M_2 (1-ch 3α) → M_3 (+${}^8Be(2^+)$) → M_4 (+direct 3α) → M_5 (+γ) → M_6 (+inelastic) → 12-channel model. Each added channel successively improves χ^2 and converges toward Sikora & Weller (2016) values [3]. Constructive interference between ${}^8Be(0^+)$ and ${}^8Be(2^+)$ pathways is the primary mechanism generating the observed asymmetric $\sigma_{3\alpha}$ lineshape near 2.65 MeV, absent in all single-channel models [4, 5].

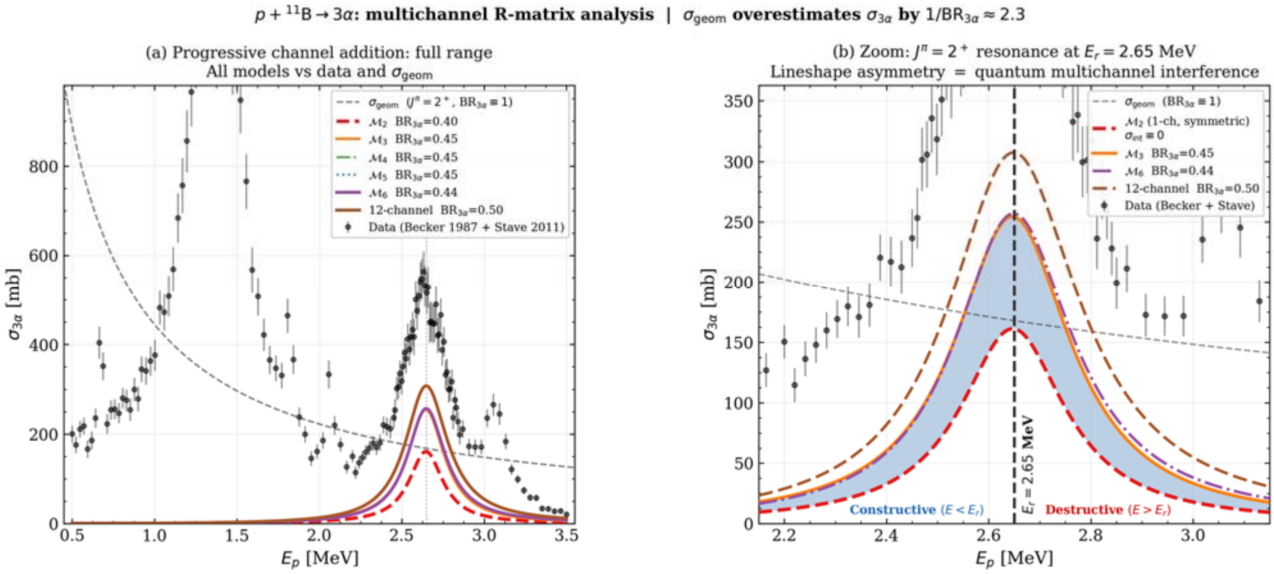

Fig. 1. (a) All models M_2–M_6 and 12-channel overlaid: $\sigma_{3\alpha}(E)$ [mb] vs energy, with data (Becker 1987 + Stave 2011 [4, 5]) and geometric limit σgeom ($J^\pi = 2+$, $BR_{3\alpha} \equiv 1$, dashed). All multichannel models lie below σgeom ($BR_{3\alpha} < 1$). (b) Zoom on the 2.65 MeV resonance: M_2 (dashed, symmetric Lorentzian, $\sigma_{int} \equiv 0$) vs M_3, M_6, 12-ch (asymmetric). Blue/red shading: constructive/destructive interference zones generated by adding the ${}^8Be(2+)$ exit channel. The lineshape asymmetry is absent in M_2 and cannot be reproduced by any single-channel model.

The constructive interference zone ($E < E_r$) enhances $\sigma_{3\alpha}(E)$ by up to 30–60 % in the sub-resonance region that dominates the Gamow integral at fusion-relevant temperatures T ~ 100–400 keV; the net effect is a direct improvement of the thermally averaged reactivity $\langle\sigma v\rangle(T)$ and a corresponding reduction of the minimum Lawson product: $\sigma int(E < E_r) > 0 \Rightarrow \langle\sigma v\rangle_{3\alpha}(M_6) > \langle\sigma v\rangle_{3\alpha}(M_2) \Rightarrow (n\tau)_{min}(M_6) < (n\tau)_{min}(M_2)$.

The corrected $\sigma_{3\alpha}$ propagates through $\sigma_{3\alpha} \rightarrow \langle\sigma v\rangle_{3\alpha}(T) \rightarrow (n\tau)_{min}$. The geometric estimate overestimates the thermally averaged reactivity $\langle\sigma v\rangle_{3\alpha}(T)$ by a factor 1.3–2.3 (largest at $T < 200$ keV). Fig. 2 shows the resulting minimum Lawson product $(n\tau)_{min}$: M_2 — 3.0×10^{23} m^{-3}s; M_6 — 1.7×10^{23} m^{-3}s; 12-channel — 1.3×10^{23} m^{-3}s [6]. The ratio $F \approx 0.6 = (n\tau)_{min}(M_6)/(n\tau)_{min}(M_2)$ provides a concrete quantitative correction: within the $J^\pi=2^+$ model hierarchy, $F < 1$ because constructive interference raises the partial $\sigma_{3\alpha}(J^\pi = 2^+)$ in M_3 vs M_2. The physically critical comparison is to the geometric limit ($BR_{3\alpha} \equiv 1$): the true $BR_{3\alpha} \approx 43\%$ implies the geometric estimate overestimates $\sigma_{3\alpha}$ and thereby underestimates $(n\tau)_{min}$ by a factor ≈2.3.

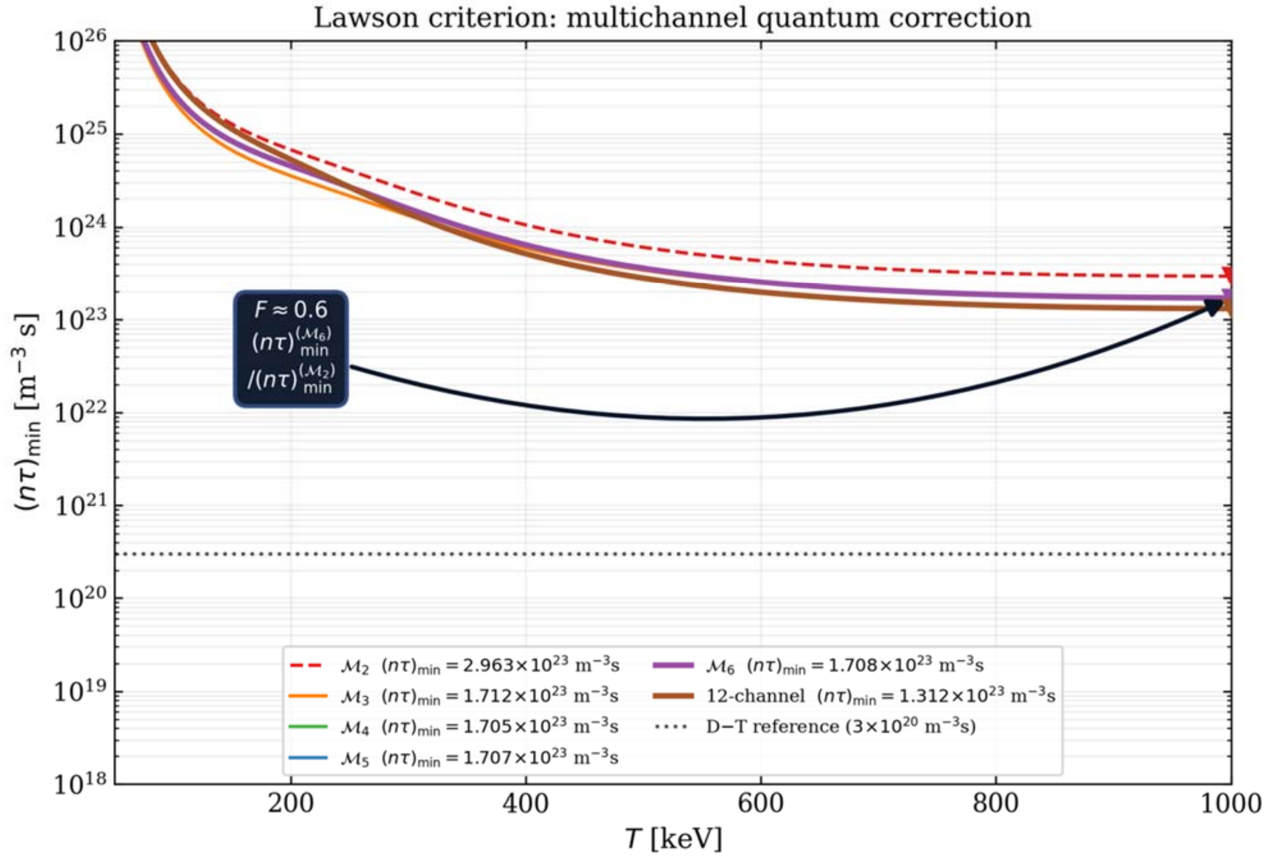

Fig. 2. Minimum Lawson product $(n\tau)_{\min}(T)$ [$m^{-3}s$] for M_2–M_6 and the 12-channel extension. Downward triangles: optimal T*. Dotted: D–T reference ($\approx 3\times10^{20}$ $m^{-3}s$). $F \approx 0.6 = (n\tau)_{\min}(M_6)/(n\tau)_{\min}(M_2)$.

The HB11 Energy laser experiment of 2022 [1] recorded α-particle flux $\sim 10^{10}$ α/sr — some 4 orders of magnitude below break-even [6]. Using σ_{geom} instead of $\sigma_{3\alpha}(M_6)$ widens this gap by a factor F ≈ 1.3–2.3. The methodology generalizes naturally to d^3He, p^6Li, $^3He^3He$, p^7Li.

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