ROTATING NEUTRON STARS WITHIN THE DEFORMED EFFECTIVE SURFACE APPROXIMATION

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In this report we present the macroscopic model for neutron stars (NSs) described as a perfect cold finite fluid system at equilibrium within the Tolman-Oppenheimer-Volkoff (TOV) assumptions modified by the Kerr and Hartle&Thorne results for a slow azimuthal angular frequency ω around the symmetry axis [1]. We take into account the NS surface deformation within the leptodermic approximation $a/R \ll 1$, where a is an NS crust thickness and R is its mean finite effective radius. Introducing the dimensionless frequency, $\omega \approx \Omega/M \approx I/M^2$, where Ω is the Kerr rotational parameter, I is the NS angular momentum, and M is its mass $(c = G = 1)$, one can use the linear perturbation approach, $\omega \ll 1$, to derive the GRT Kerr metric in the spherical coordinates outside $r > R$ and inside $r < R$ of the NS. The spacetime interval is written as

$$ds^2 = e^{\nu} dt^2 - 2\tau\Omega sin^2\theta dt d\varphi - e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\varphi^2, \qquad (1)$$

where $\nu(r)$ and $\lambda(r)$ are the Schwarzschild metric functions at the zero order over ω. Therefore, one obtains $\tau = X(x)Y(\theta)$, where $Y(\theta) = F(\alpha, \beta; 2; w)$ is the hypergeometric function of the following arguments: $\alpha = \left[3 - \sqrt{9-q}\right]/2, \beta = \left[3 + \sqrt{9-q}\right]/2, w = cos^2(\theta/2)$, and q is the separation variable constant. Here, $x = r/R_S$ for $r < R$ and $R_S = (3/8\pi\rho_0)^{1/2}$ is the Schwarzschild radius, ρ_0 is the inner statistically averaged mass density larger than or of the order of that for nuclear matter, and $x = r/r_g$ for $r > R$, where $r_g = 2M$ is the gravitational radius. For the leading outer solution ($r > R$), one obtains $X_{out}(x) = x^{(1-\sqrt{9-4q})/2}$ up to terms of relatively smaller order of $1/x$ for $q < 2$. For large x, one has the Boyer&Lindquist limit r_g/r. For the inner solution ($r < R$), one obtains approximately $X(x) = c_1 J_p(\sqrt{5}x)$, where J_p is the Bessel function with the index $p = (9 - 4q - 6R^2/R_S^2)^{1/2}$. The constant $c_1 = X_{out}\,(R^2/R_S^2)/J_p\,(\sqrt{5}\,R/R_S)$ is determined by the boundary condition at $r = R$, $r_g/R \approx R^2/R_S^2$. The surface gradient terms are taken into account through the energy density $\varepsilon(\rho) = \rho + C(\nabla\rho)^2$, where C is the interparticle interaction constant for the macroscopic equation of state (EoS) in the Extended Thomas Fermi (ETF) approach but with accounting for a strong gravitation. This constant including also a gravitational field beyond the nuclear forces is related to the leptodermic parameter by $a/R = (C\rho_0 K)^{1/2}$, where K is the total nuclear-gravitational incompressibility [1].

The angular momentum I and the moment of inertia (MI), $\Theta = dI/d\,\omega$, are macroscopically calculated in the adiabatic $\omega \ll 1$ approximation. The adiabatic MI can be expressed as $\Theta = \Theta_{av}/\left(1 - \Theta_{t\varphi}\right)$ in terms of the statistically averaged Θ_{av}, and the correlation $\Theta_{t\varphi}$ contributions by involving the time–angle t, φ gravitational coupling (see (1)). The correlation term $\Theta_{t\varphi}$ becomes significant for a strong NS gravity, in contrast to the statistically averaged MI Θ_{av} neglecting t, φ correlations . The MI contributions Θ_{av} and $\Theta_{t\varphi}$ are the sums of the volume and surface components obtained through the ETF energy density $\varepsilon(\rho)$, $\Theta_i = \Theta_{iV} + \Theta_{iS}$ $(i = av, t\varphi)$, and for the total energy, one has $E = \int \varepsilon(\rho)\, dV = E_V + E_S$, where the subscript V shows the NS volume contribution ($E_V \propto V$), and S means the NS surface area ($E_S = \sigma S$, where $\sigma \propto a\rho_0 K J(R)$ is the tension coefficient, and $J(R) = 1/(1 - R^2/R_S^2)^{1/2}$ is the Jacobian $J(r)$ for the radial variable transformation through the NS surface at $r = R$).

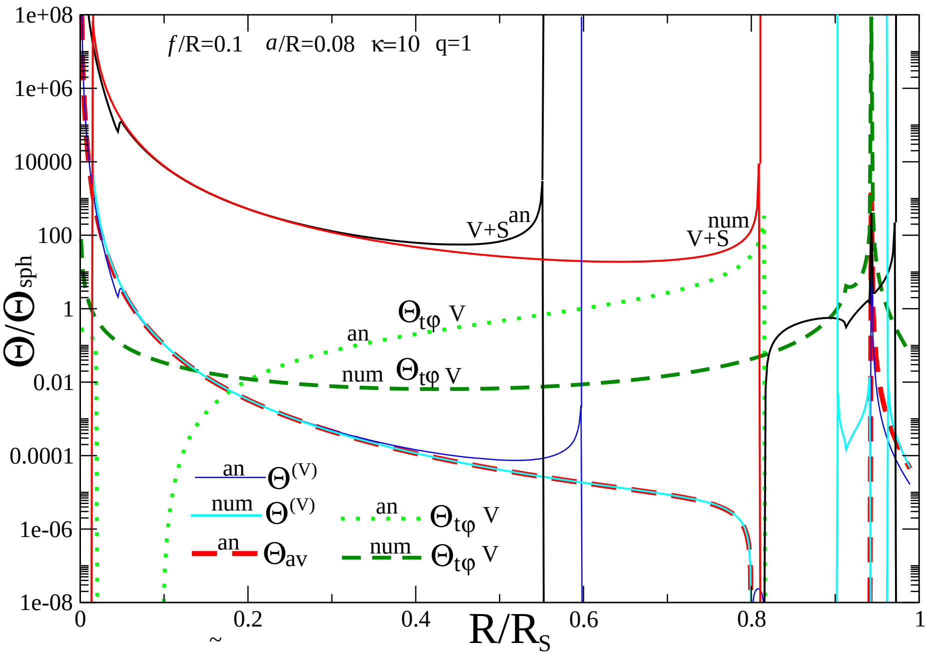

Figure 1. ... *heir statistically averaged ("av"), correlational ("tφ"), and the volume ("V") contributions.*

Figure 1 shows the comparison of the full analytical ("an") and numerical ("num") NS MI, and its volume (V) contributions as well their statistical (Θ_{av}) and correlational MI ($\Theta_{t\varphi}$) parts for an NS with its deformed surface as functions of the effective radius variable R in units of R_S, R/R_S . In these calculations, the dimensionless incompressibility $\kappa = K/12m = 10$, where m is the test particle mass, for a strong gravity ($\kappa = 2$ for the small gravity case), and for a small leptodermic parameter, $a/R = 0.08$, are used. For small NS surface deformations, one can approximate them by spheroidal shapes with the focus distance f in units of the radius R given by, $f/R = (a - b)/R = 0.1$ where a and b are the spheroid semi-axes. For example, the separation constant value for the symmetry breaking is given by $= 1$. The correlation contribution $\Theta_{t\varphi}$ to the MI has a minimum at $R/R_S \approx 0.3$. The $\Theta_{t\varphi}$ is increased asymptotically with further increasing of R/R_S. As the main result, one obtains the rotational constraints, $R_1 < R < R_2 < R_S$, from both sides because of the two poles at $R_1/R_S \approx 0.015$ and $R_2/R_S \approx 0.82$ of the total MI Θ (red solid) with accounting for the surface contribution, and due to a strong gravity. For $q \leq 0$, in particular for the spherically symmetric $q = 0$ case, $R_1 = 0$. The gravitational effect is strong because of the correlation and surface contributions for the incompressibility values of the order of or larger than $\kappa = 10$. A large surface contribution can be seen from the figure, cf. black and thin blue solid curves.

As perspectives, our analytical macroscopic approach can be generalized to the asymmetric deformed rotating systems including the isotopic two-component symmetry energy into EoS like for neutrons and protons in nuclei. We are planning also to derive the rotational corrections to the TOV equations for the pressure calculations through the NS surface.

1. A.G. Magner et al. Rotating neutron stars within the macroscopic effective-surface approximation 2025. arXiv: 2509.13129 [gr-gc]. Nucl. Phys. At. Energy 27(1) (2026) 5. Submitted to the J. Phys. G (2025).