# THEORY AND EXPERIMENT OF GRAVITON PHYSICS

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# Plan of the talk

1. Experimental searches for graviton mode;
2. Microscopic theory with Hamiltonians at different filling factors;
3. The hierarchical relationship between the microscopic Hamiltonians;
4. The twisted K-group for calculation of Hilbert space states of charged particles;
5. Conclusions.

# Experimental searches for graviton mode

**Gravity is one of the fundamental interactions in nature, along with electromagnetic, strong and weak interactions. A full-fledged theory of quantum gravity has remained elusive for many decades, but if gravity is quantum in nature, there should be a weak excitation of the gravitational field, the graviton. Scientists have presented the first experimental data confirming the existence of a chiral graviton mode, which should help us to understand how gravity works. The authors of the study from the US, Germany and China found these particles in a special type of liquid that behaves in a special way under a magnetic field.**

# Microscopic theory with Hamiltonians at different filling factors

- **In the condensed state of matter, excitations can arise that obey the same equations as the quanta of some fundamental fields.**
- **These are, as a rule, very special conditions that require the presence of low temperatures, a magnetic field, and in some cases also the limitation of two dimensions (in thin films or, under certain conditions, on the surface of the sample).**
- **Two-dimensional “universe" connected with the quantum Hall effect, is characterized by Fermi velocity of the chiral Luttinger liquid of the edge transport and by magnetic length. modes, called graviton modes (GM), arise in the quantum fluid of electrons under the influence of a strong magnetic field at low temperatures.**
- **This effect is called the fractional quantum Hall effect (FQHE) in a two-dimensional fluid. To study gravitational modes, inelastic scattering of photons is considered, modeled using microscopic theory with Hamiltonians at different filling factors.**

# Hamiltonians at different filling factors

We’ll use the following Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \frac{1}{2m} \bar{g}^{ab} \hat{\pi}_{ia} \hat{\pi}_{ib} + \hat{V}_{int}, \quad (1)$$

where $\hat{\pi}_{ia} = \hat{p}_{ia} + e\hat{A}_{ia}$ – the dynamical momentum operator of the i-th electron, $A_i$ is the external vector potential, connected with magnetic field by formula $B = \epsilon^{ab}\partial_a A_{ib}$. $\hat{V}_{int}$ describes the dynamics only within a single LL, the magnetic length is $l_B = \sqrt{1/eB}$. The Hilbert space of a single LL, referred to as the lowest LL (LLL), is parametrized by the metric $\bar{g}^{ab}$, which leads to density modes in higher LLs, known as “cyclotron gravitons".

# The dynamics

The dynamics is determined by the guiding center coordinates $\bar{R}^a = \hat{r}^a - \epsilon^{ab}\hat{\pi}_b l_B^2$. The interaction energy in (1), $\hat{V}_{int}$, is a functional of $\bar{R}^a$ and expressed as

$$\hat{V}_{int} = \int d^2q\, V_{|q|} \bar{\rho}_q \bar{\rho}_{-q},$$

with $\bar{\rho}_q = \sum_i e^{iqR_i}$ – the guiding center density operator and $|q| = \sqrt{\bar{g}^{ab} q_a q_b}$ – the distance in the momentum space. There is presented the analogy to the “cyclotron graviton" in the integer quantum Hall effect through the type of gravitational interaction, $\hat{V}_{int} = \hat{V}_1^{2body}$. The Hilbert spaces like LLL are called conformal Hilbert spaces (CHSs) as they are generated by the conformal operators like the Virasoro algebra, known as the Virasoro constraint in string theory, applied only on the physical states [4]. So such CHSs are built up with quasiparticles.

# Multiple GM

To study multiple GM, it is necessary to study the properties of the energy levels of electron-electron interaction described by the CHS. The CHS of a lowest LL, LLL is parametrized by the unimodular metric $\bar{g}^{ab}$, quantum fluctuations of which lead to density modes in higher LLs, “cyclotron gravitons". The energy of GM is very high, with a large magnetic field. These GM are connected with higher energy levels within integer quantum Hall effect (IQHE), since the LLL is fully filled. For FQHE in a partially filled LL, the dynamics is determined by the guiding center coordinates $\bar{R}^{a}$, which determine the interaction energy $\hat{V}_{int}$. Such Hilbert spaces are spanned by zero energy many-body states of special local Hamiltonians, including the pseudopotentials. Fig. 1 illustrates a hierarchical structure of CHS in the LLL.

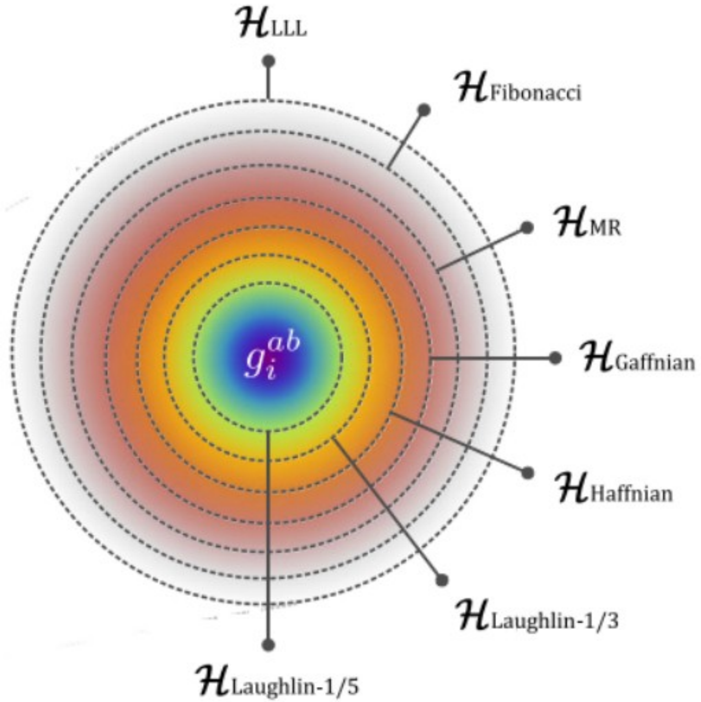


**Fig. 1**. *The hierarchical structure of the CHSs in the LLL.*

So, the subspaces of the LLL have a hierarchical structure $H_{k+1} \subset H_k$ . The rest of the cyclotron GMs are examples to understand the emergence of multiple GMs from $V_{int}$ in accordance with the hierarchical structure. To understand that the number of GMs is a dynamic property, we'll consider the model Hamiltonian for the GMs

$$\hat{V}_{int} = \sum_{k=1}^{m} \lambda_k \hat{V}_k \, . \qquad (2)$$

The inelastic scattering with the circularly polarized light, as was shown in [5] for the Jain states with a filling factors ν = 2/7, 2/9, 1/4 and three body components of interaction (2), provides an example to understand gravitons' dynamics with peaks observed at almost the same frequency for each polarization in the second LL, SLL. So an introduction of a third component, $V^{3bdy}{}_9$, to the microscopic Hamiltonian can lead to the splitting of the two GMs to three GMs. Moreover, it is interesting to stress the hierarchical relationship, $H_9^{3bdy} \subset H_1^{2bdy} \subset H_8^{3bdy}$ .

# The twisted K-group

We can use the apparatus of the twisted K-group for calculation of Hilbert space states of charged particles for explanation of the FQHE, which are topological phases of LLLs. As for the FQHE we are dealing with a set of levels with a filling factor less than one. The lowest LL is parametrized by the metric $\bar{g}^{ab}$ with a hierarchical structure of CHS as presented in Fig. 1. Since we are dealing with four types of interaction, it is appropriate to use the apparatus of vector bundles to describe a complex formation of D-brane type. B-field interacting with D-branes can be taken into account through the Dixmier-Douady invariant, which characterizes the bundles and describes the strength of the Neveu-Schwarz B-field interacting with D-branes. D-branes are topological solitons whose charges are described by Grothendieck K-groups. One of the most exciting discoveries of D-brane theory is the prediction of the non-commutativity of space-time coordinates [6]. The description of this non-commutative geometry is realized in terms of the twisted K-theory of C*-algebras. Let us calculate the topological charges of the D6-brane using twisted K-theory methods. Consider the following vector bundle over a compact manifold X:

# The twisted K-group

$$\begin{array}{ccc} F & \rightarrow & E_H \\ & & \downarrow \\ & & X \end{array} \qquad [H] \notin \mathrm{Tors}(H^3(X, Z))$$

H– the Dixmier-Douady invariant, describing the intensity of the Neve-Schwartz B-field

characterizing the bundles $E_H$ .

Reduction of twisted K groups to an exact sequence of the form

$$0 \rightarrow Z \rightarrow Z \rightarrow Z_n \rightarrow 0$$

leads to the result

$$K_0(S^3, n[H])=Z_n$$

# The twisted K-group

This group value determines the topological charges of the D6-brane in the presence of the Neveu-Schwarz -field. Since the theory of relativity is defined in four-dimensional space, three coordinates plus time, the three-dimensional sphere with the punctured point is an approximation of three-dimensional Euclidean space. The ten-dimensional space in string theory, compactified onto six-dimensional space, is described in terms of a topological invariant of D6-brane, in this case by the twisted K-group, which takes values in the cyclic group of order n, $Z_n = \{0, 1, ..., n - 1\}$. We see that with the addition of the next interaction component to formula (2), a new energy level or a new excited state of the GM is added, which indicates a consistent increase in the number of the energy levels within the FQHE. The nesting of each previous level into the next one is fixed, $H_1 \subset H_2 \subset H_3 \subset ... \subset H_n$, which is well reflected by the value of the twisted K-group. Thus, the values of the topological charges of the K-group give us information about the nature and number of energy levels that are excitations of the LLL within the FQHE framework.

# CONCLUSIONS

- We have considered the fractional quantum Hall effect associated with graviton modes that arise in the quantum fluid of electrons under the influence of a strong magnetic field at low temperatures;
- We presented the geometric origin of GM and the hierarchical structure of conformal Hilbert spaces as null spaces of model Hamiltonians. So, the subspaces of the LLL have a hierarchical structure $H_{k+1} \subset H_k$;
- The inelastic scattering with the circularly polarized light, provides an example to understand gravitons' dynamics. So an introduction of a third component to the microscopic Hamiltonian can lead to the splitting of the two GMs to three GMs. So, it is interesting to stress the hierarchical relationship,

$$H_9^{3bdy} \subset H_1^{2bdy} \subset H_8^{3bdy}$$

- Using the twisted K-group apparatus for calculation of Hilbert space states of charged particles we received

$$K_0(S^3, n[H]) = Z_n$$

which indicates a consistent increase in the number of the energy levels within the FQHE.

# THANK YOU