BENT CRYSTALS FOR BEAM STEERING IN ACCELERATORS

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Academician of the National Academy of Sciences of Ukraine Prof. Mykola Shul’ga

З 1996 р. Микола Федорович Шульга (15.09.1947–23.01.2024) очолював ІТФ ім. О.І. Ахієзера

З 2015 р. по 2024 р. Академік-секретар Відділення ядерної фізики та енергетики НАН України

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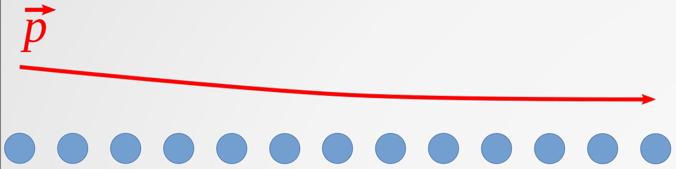
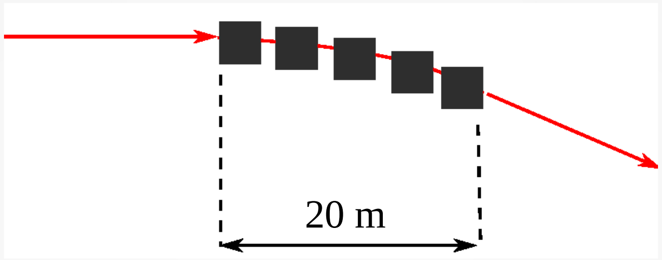
Bent crystals and magnetic deflection systems

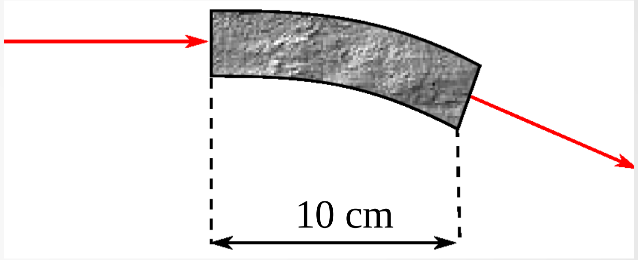


$E_{str} \sim 10^{10}$ V/cm
$E_{pl} \sim 10^{9}$ V/cm

Magnets

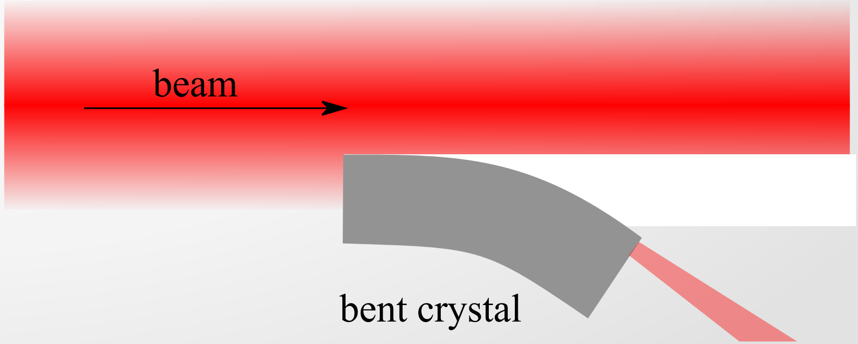


Bent crystal



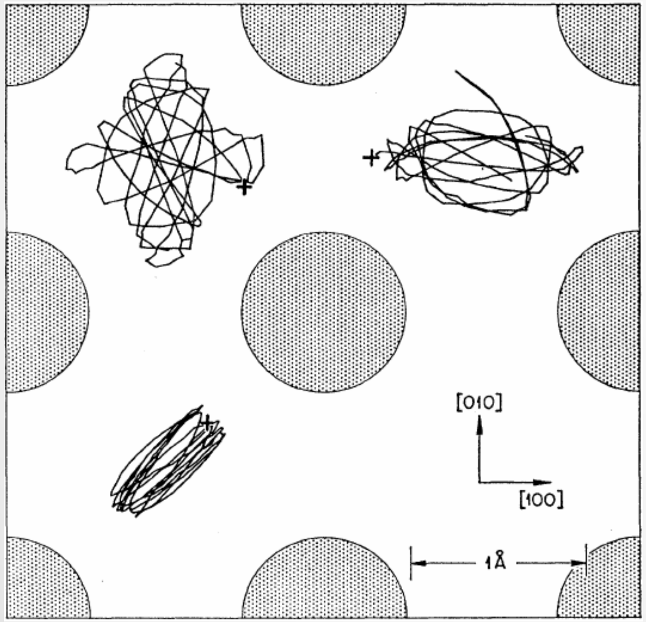
Advantages of bent crystals in comparison with magnetic deflection systems:

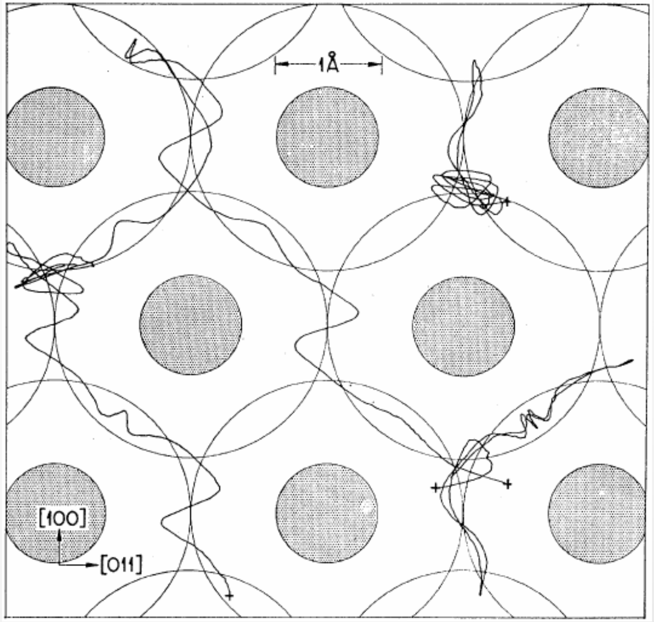
- Small size
- do not need electricity consumption
- do not need cooling



Розсіювання заряджених частинок у кристалі

Robinson M. T., Oen O. S., Holmes D. K. Computer studies of anomalous penetration of Cu recoil atoms in Cu crystal. Proc. of Conference «Bombardment Ionique». CNRS. Paris. 1962. P. 105.
Robinson M. T., Oen O. S. Computer studies of the slowing down of energetic atoms in crystals. Phys. Rev. 1963. Vol. 132, No. 6. P. 2385.





Cu^{+}, E=1-10 кэВ

Lindhard J. Influence of crystal lattice on motion of energetic charged particles. Mat. Fys. Medd. Dan. Vid. Selsk. 1965. Vol. 34, No. 14. P. 1–64.

Наближення безперервного потенціалу

$$\frac{d}{dt}\frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} = -q\nabla\Phi_c(\mathbf{r})$$

$$\Phi_c(\mathbf{r}) = \sum_n \Phi_a(\mathbf{r}-\mathbf{r}_n)$$

$$\Phi(\boldsymbol{\rho}) = \frac{1}{L}\int_{-\infty}^{\infty} dz\,\Phi_c(\boldsymbol{\rho}, z)$$

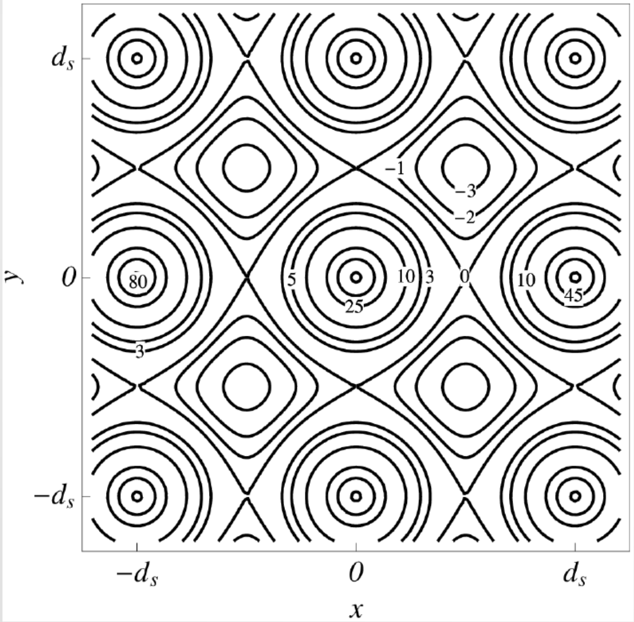
$$\ddot{\boldsymbol{\rho}} = -\frac{c^2 q}{E_\parallel}\frac{\partial}{\partial\boldsymbol{\rho}}\Phi(\boldsymbol{\rho})$$

$$E_\parallel = c\sqrt{p_\parallel^2+(mc)^2}$$

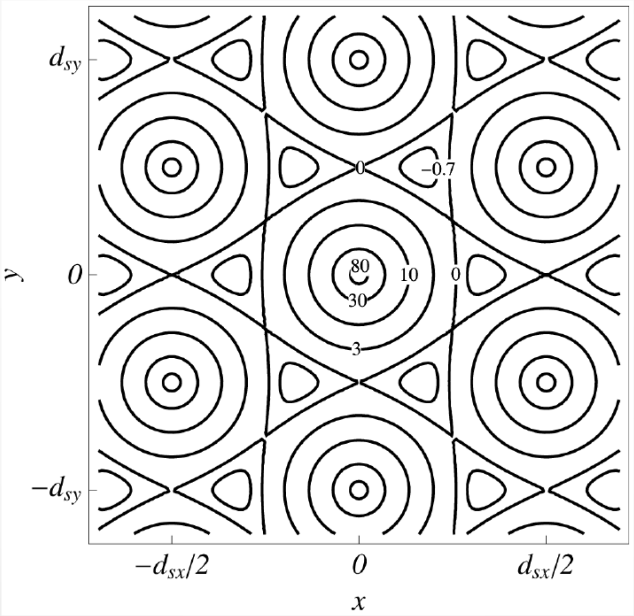
Lindhard J. Influence of crystal lattice on motion of energetic charged particles. Mat. Fys. Medd. Dan. Vid. Selsk. 1965. Vol. 34, No. 14. P. 1–64.

Potential of crystal atomic strings

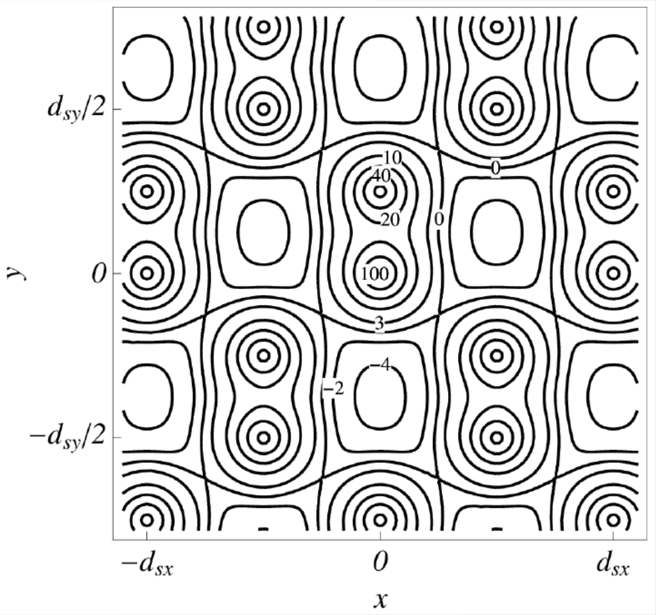
Si <100>



Si <111>

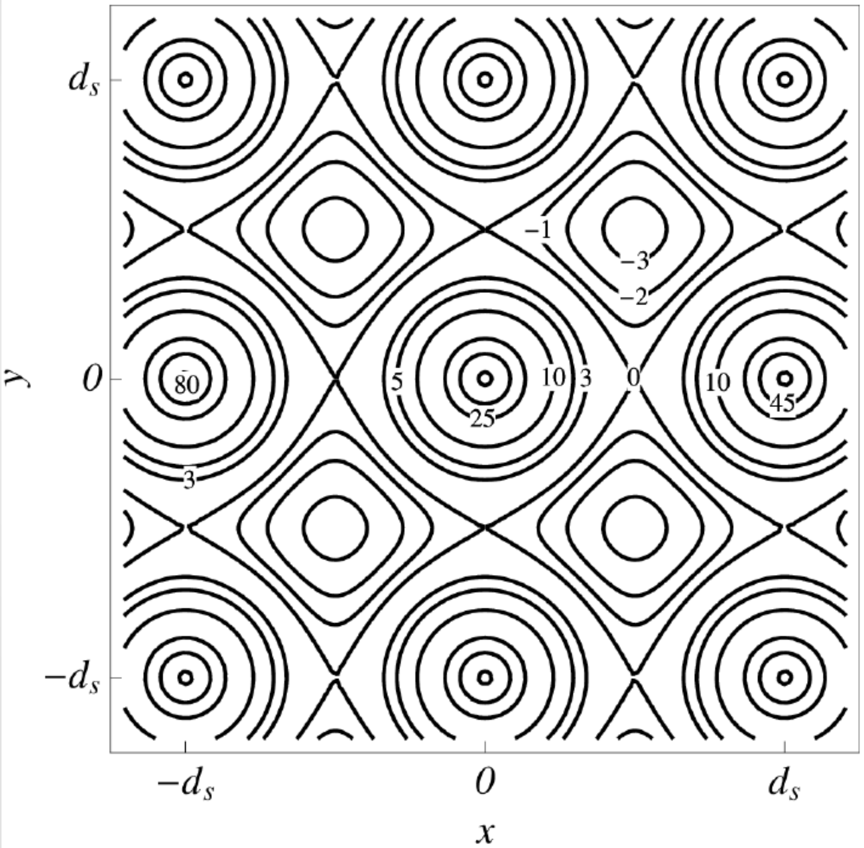


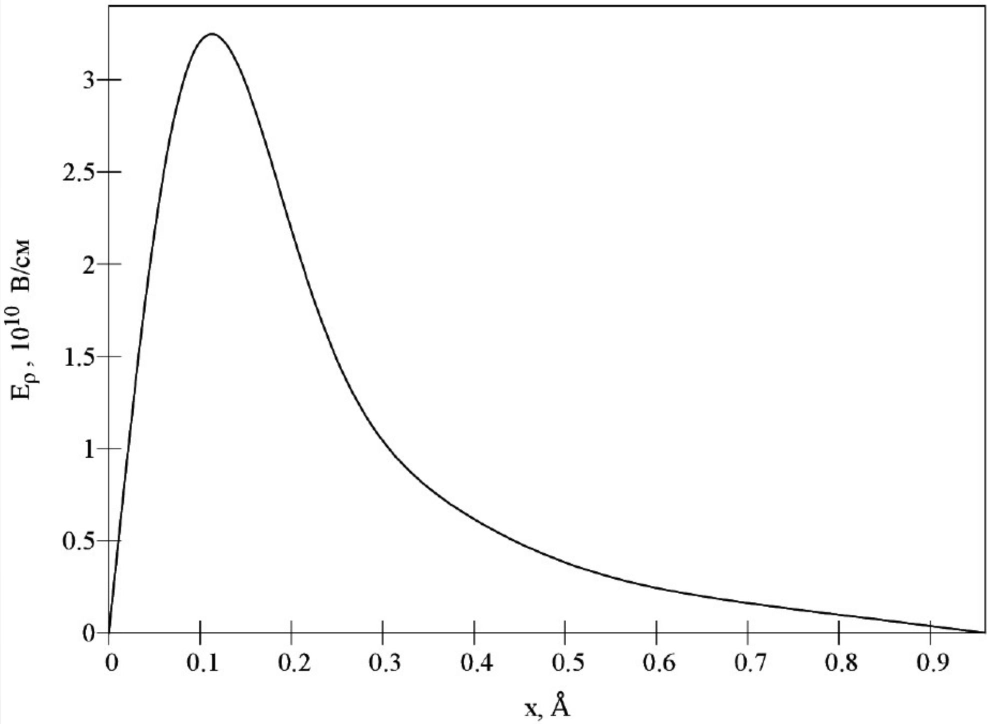
Si <110>



Potential of crystal atomic strings

Si <100>

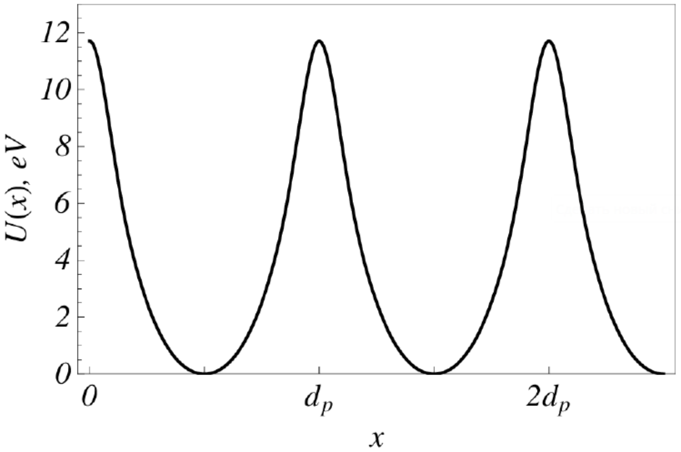


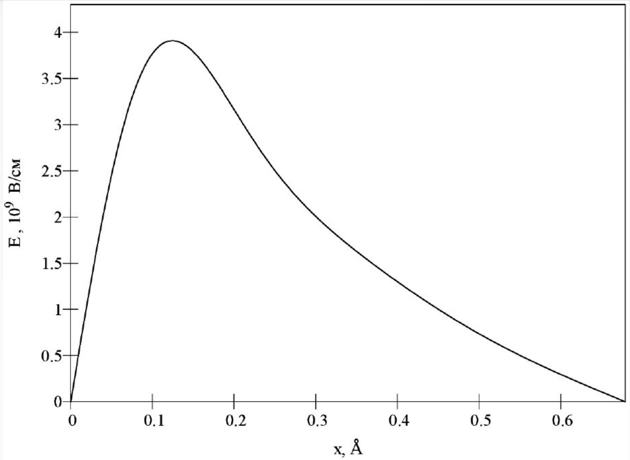


$E_{str} \sim 10^{10}$ V/cm

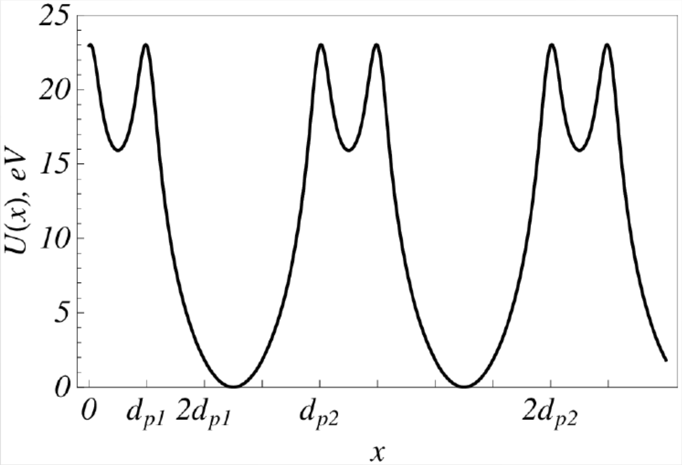
Potential of crystal atomic planes

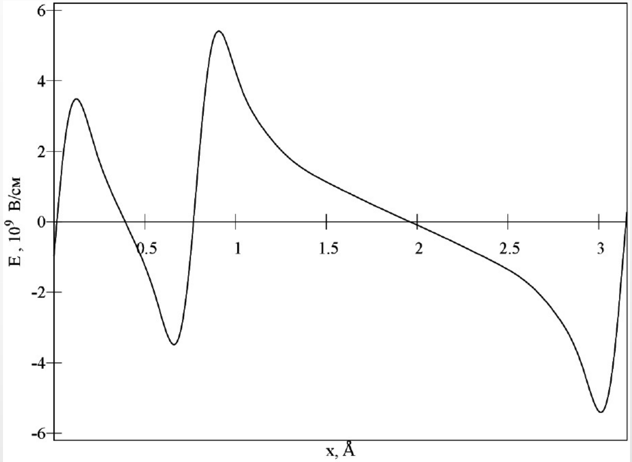
Si (100)





Si (110)



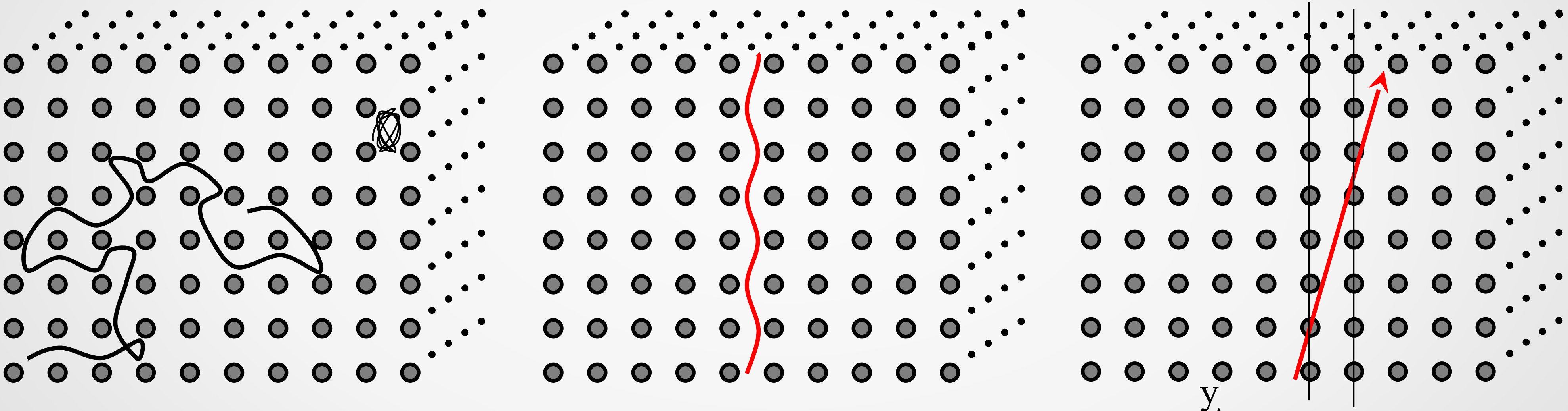


E_{str} ~ 10^{10} V/cm

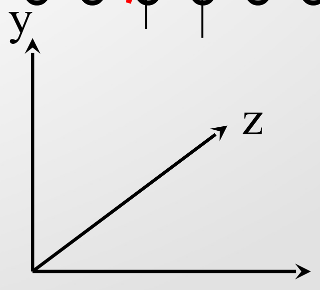
E_{pl} ~ 10^9 V/cm

Regimes of motion in a ctystal

$\psi_x \approx \psi_y < \psi_c$ $\qquad$ $\psi_x < \theta_c\,,\ \psi_y \gg \psi_c$ $\qquad$ $\psi_x > \theta_c\,,\ \psi_y \gg \psi_c$

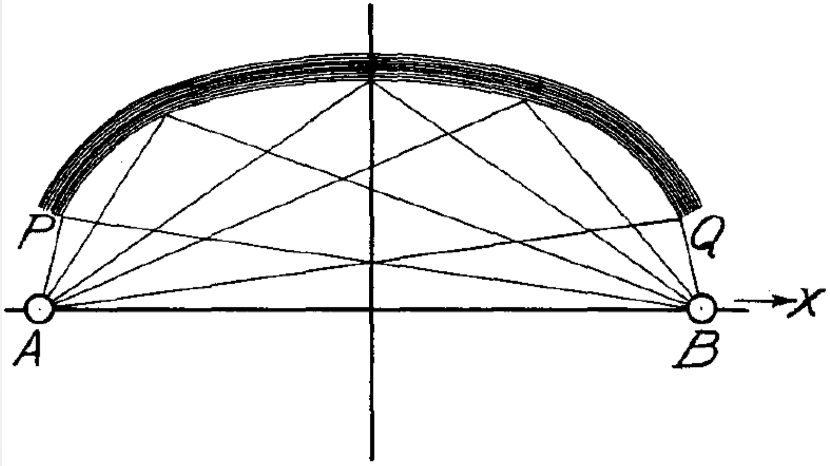


$$v_z \approx c\,, \qquad \psi_x = \frac{v_x}{c}\,, \qquad \psi_y = \frac{v_y}{c}\,, \qquad \psi_c \approx 2\,\theta_c \sim 10^{-5}$$

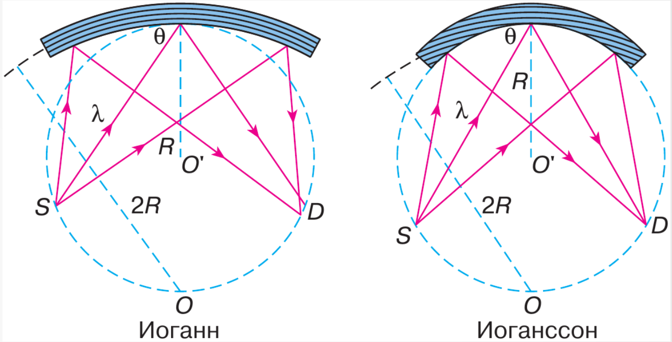


Вигнуті кристали в рентгенівській спектроскопії

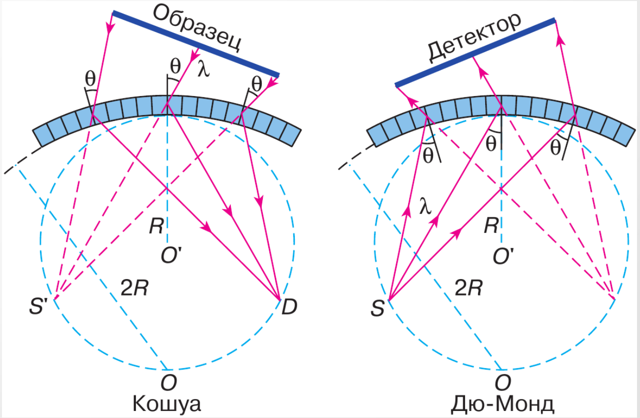
DuMond J. W., Kirkpatrick H. A. The multiple crystal x-ray spectrograph. Rev. Sci. Instrum. 1930. Vol. 1, No. 2. P. 88–105



Методи відбиття

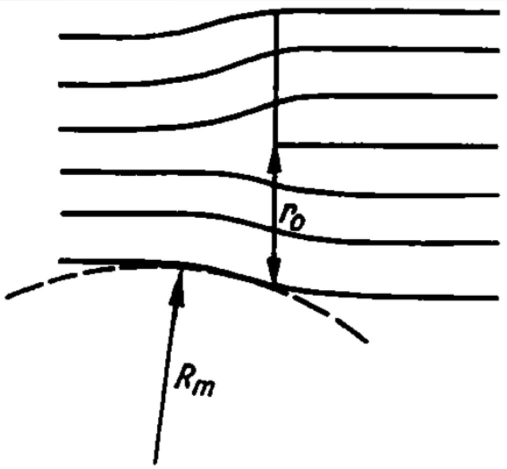


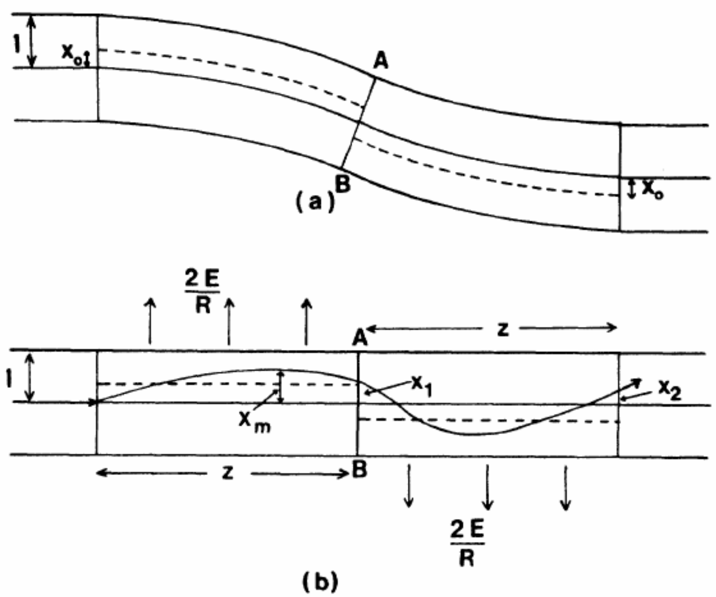
Johann H. The production-aperture X-ray spectra with the help of concave crystals. Eur. Phys. J. A. 1931. Vol. 69. P. 185.
Johansson T. Über ein neuartiges, genau fokussierendes röntgenspektrometer. Zeitschrift für Physik. 1933. Vol. 82, No. 7-8. P. 507–528.
Cauchois Y. Spectrographie des rayons X par transmission d'un faisceau non canalisé à travers un cristal courbé (I). J. phys. Radium. 1932. Vol. 3, No. 7. P. 320–336.
DuMond J. W. A high resolving power, curved-crystal focusing spectrometer for short wave-length x-rays and gamma-rays. Rev. Sci. Instrum. 1947. Vol. 18, No. 9. P. 626–638.



Методи проходження

Каналювання заряджених частинок у кристалі з дислокацією та у вигнутому кристалі





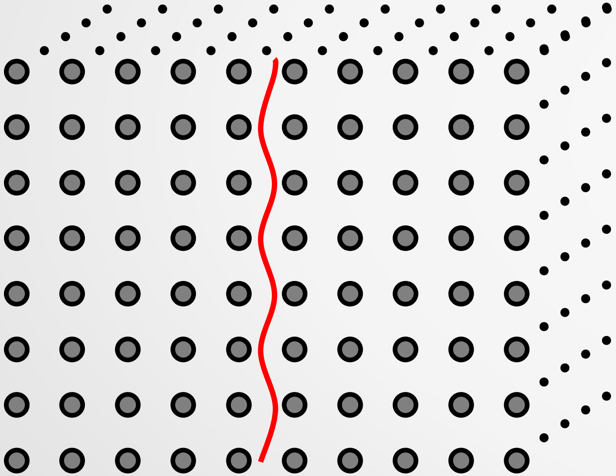
Quéré Y. Dechannelling cylinder of dislocations. Phys. Stat. Solidi B. 1968. Vol. 30, No. 2. P. 713–722.

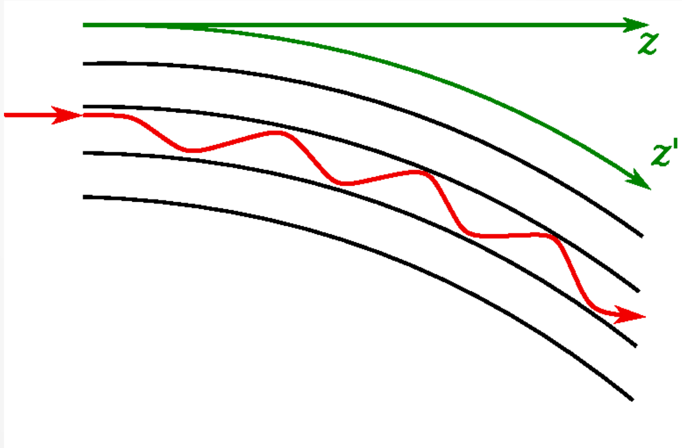
Pathak A. P. Motion of charged particles in curved planar channels: Effects of dislocations. Phys. Rev. B. 1976. Vol. 13, No. 11. P. 4688.

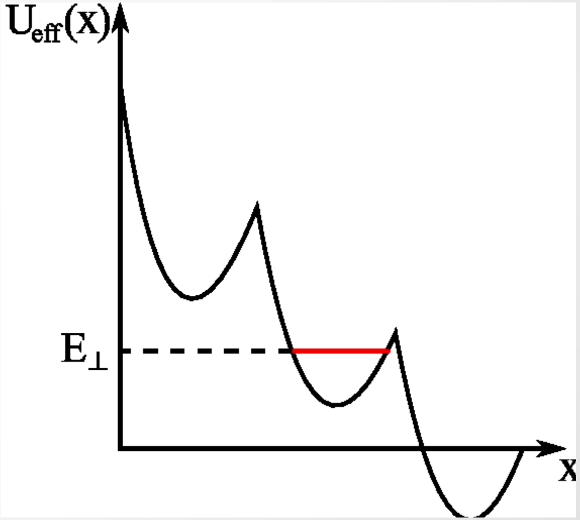
Planar channeling

Tsyganov E. N. Fermilab TM-682, TM-684. 1976.

$$\psi_x < \theta_c\,, \psi_y \gg \psi_c$$







Planar channeling

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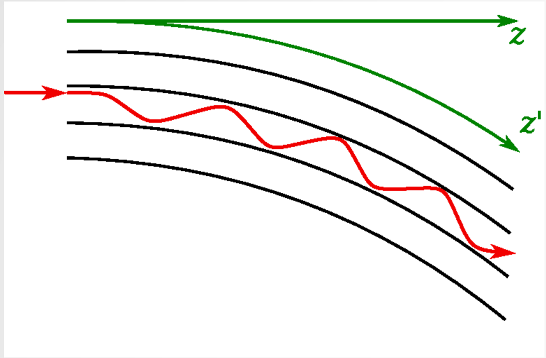
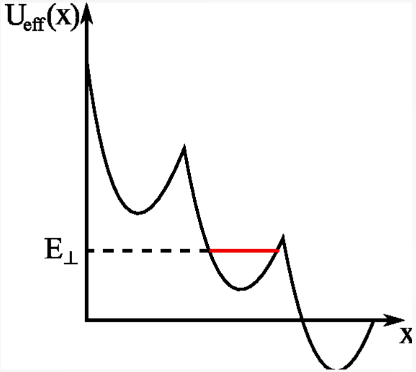
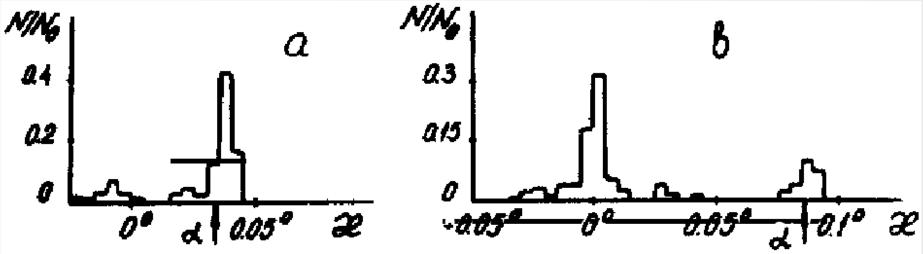
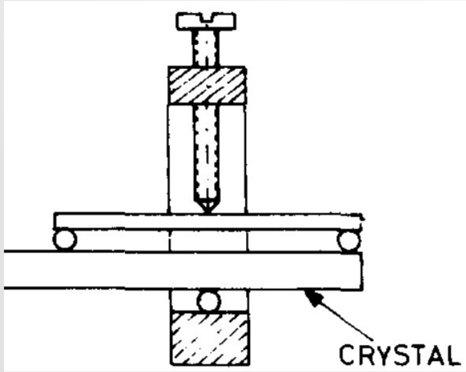


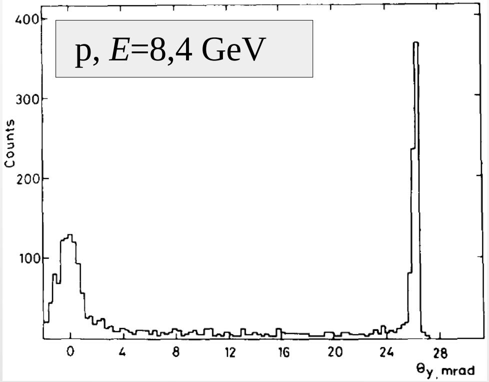


Tarantin A. M., Tsyganov E. N., Vorobiev S. A. Computer simulation of deflection effects for relativistic charged particles in a curved crystal. Phys.Lett. A. 1979. Vol. 72, No. 2. P. 145–146.



p, E=1 GeV, a) R=0,29 cm, b) R=0,112 cm





Elishev A. F., Filatova N. A., Golovatyuk V. M. et al. (I.A. Grishaev, G.D. Kovalenko, B.I. Shramenko) Steering of charged particle trajectories by a bent crystal. Phys. Lett. B. 1979. Vol. 88, No. 3-4. P. 387–391.

Вимірювання магнітного моменту короткоживучих заряджених частинок

Барышевский В. Г. Вращение спина ультрарелятивистских частиц, пролетающих через кристалл. Письма в ЖТФ. 1979. Т. 5, No 3. С. 182–184.
Любошиц В. Л. Поворот спина при отклонении релятивистской заряженной частицы в электрическом поле. Ядерная физика. 1980. Т. 31, No 4. С. 986–992.

$$\frac{d\vec{\zeta}}{d\tau} = \frac{2\mu}{\hbar}\vec{\zeta}\times\vec{H}^* - (\gamma-1)\left(\vec{l}\times\frac{d\vec{l}}{d\tau}\right)\times\vec{\zeta},$$

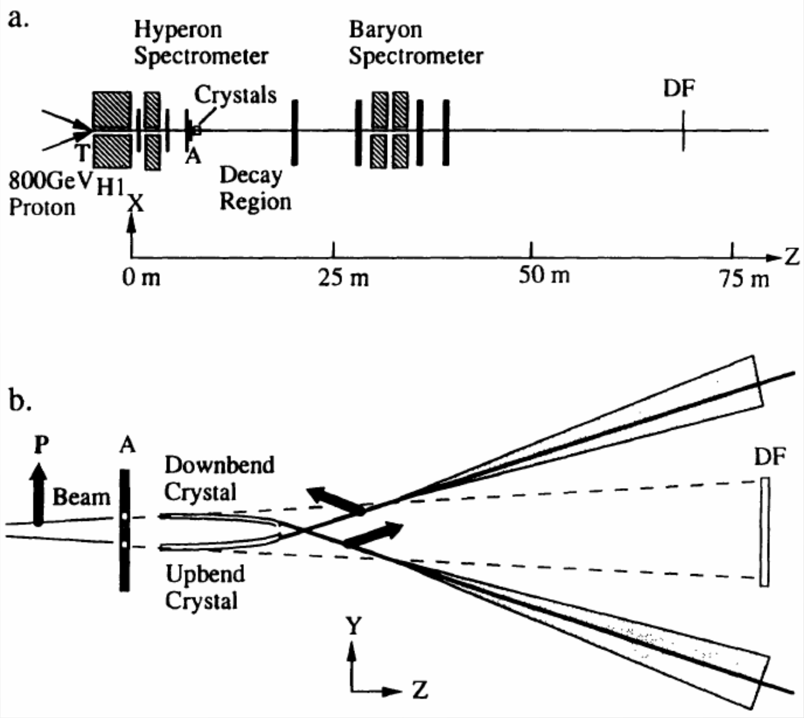
$\vec{H}^*$ напруженість магнітного поля в системі спокою частинки

$\vec{\zeta}$ подвоєне середнє значення оператора спіна частинки

Магнітний момент частинки $\mu = \frac{qg}{2mc}\hbar s$

$$\vec{H}^* = \gamma\left(\vec{H} - \frac{\vec{v}}{c}\vec{l}\times\vec{E}\right) - (\gamma-1)\vec{l}\left(\vec{l}\vec{H}\right)$$

$$\theta = \left(\frac{g-2}{2}\frac{\gamma^2-1}{\gamma} + \frac{\gamma-1}{\gamma}\right)\theta_0$$



Σ^+

Chen D. et al. First observation of magnetic moment precession of channeled particles in bent crystals. Phys. Rev. Lett. 1992. Vol. 69, No. 23. P. 3286–3289.

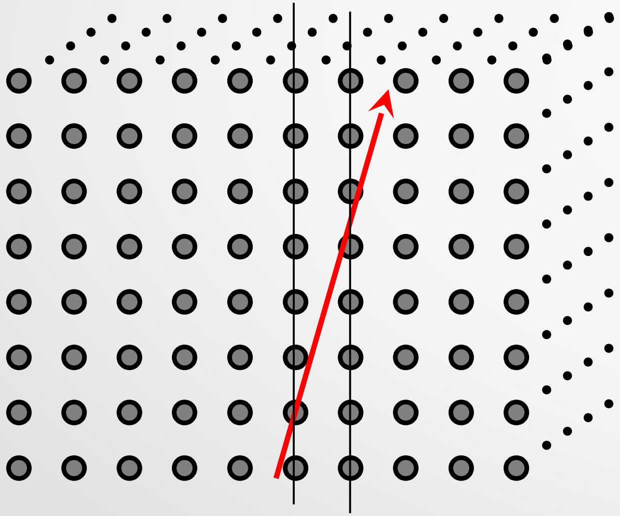
Volume reflection

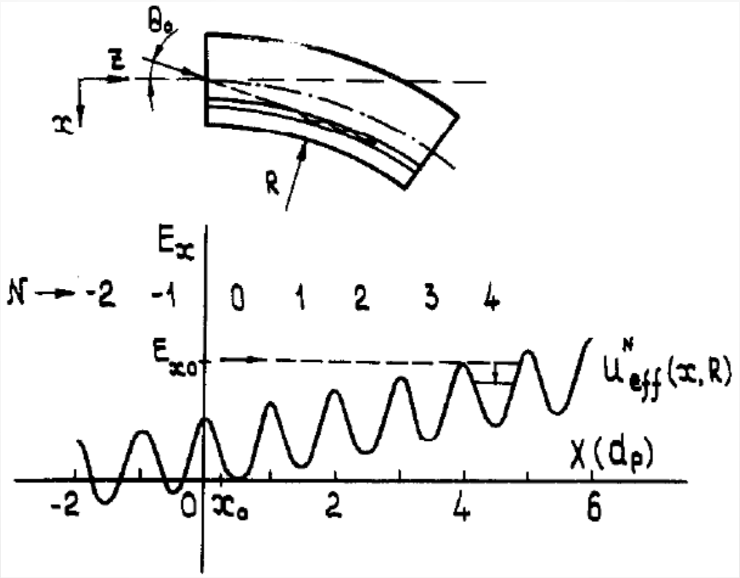
Taratin A. M., Vorobiev S. A. Phys. Lett. A. 1986. Vol. 115, No. 8. P. 398–400.
Taratin A. M., Vorobiev S. A. Nucl. Instrum. Meth. B. 1987. Vol. 26, No. 4. P. 512–521.
Taratin A. M., Vorobiev S. A. Phys. Lett. A. 1987. Vol. 119, No. 8. P. 425–428.

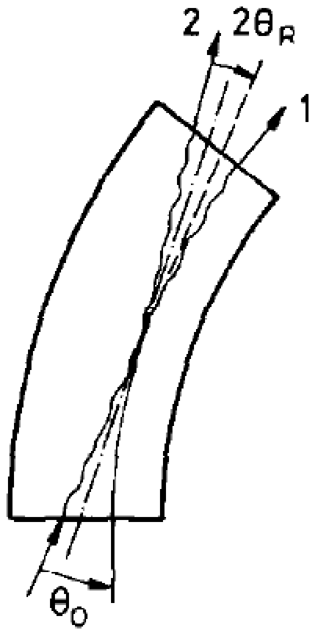
$$E_x = \frac{pv\theta_x^2}{2} + U_{eff}(x, R)$$

$$U_{eff}(x, R) = U(x) + pv\frac{x}{R}$$

$$\psi_x > \theta_c\,,\ \psi_y \gg \psi_c$$

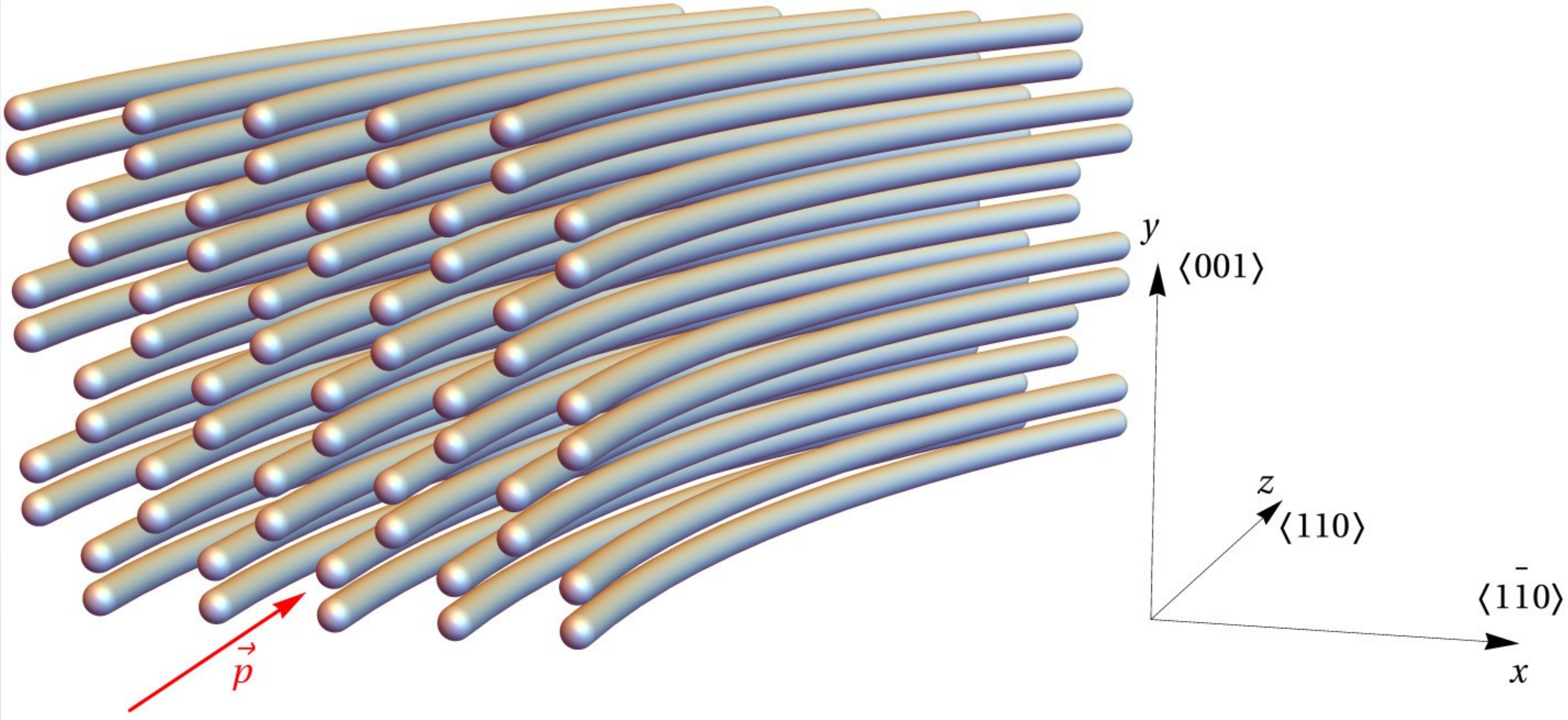






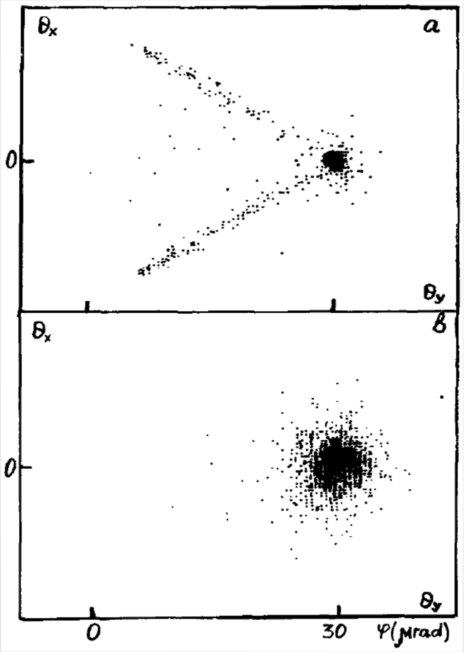
Stochastic deflection

Grinenko A. A., Shul’ga N. F. J. Exp. Theor. Phys. Lett. 1991. Vol. 54. P. 524–528.
Greenenko A. A., Shul’ga N. F. Nucl. Instrum. Meth. B. 1994. Vol. 90, No. 1-4. P. 179–182.
Shul’ga N. F., Greenenko A. A. Phys. Lett. B. 1995. Vol. 353, No. 2. P. 373–377.



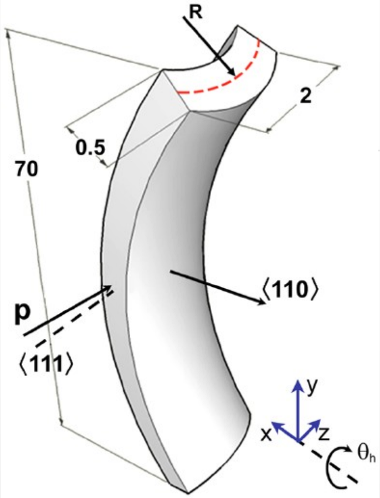
Stochastic deflection

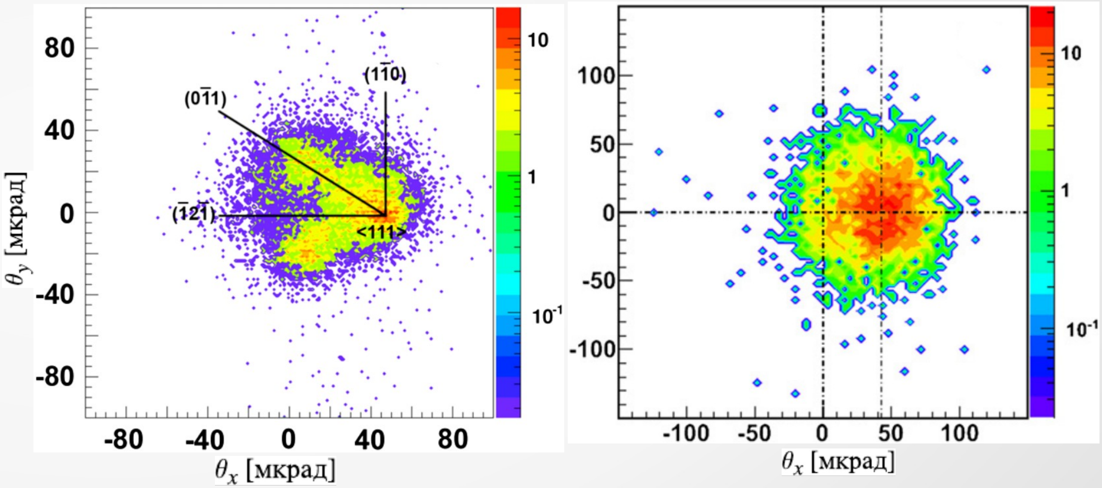
Grinenko A. A., Shul'ga N. F. J. Exp. Theor. Phys. Lett. 1991. Vol. 54. P. 524–528.
Greenenko A. A., Shul'ga N. F. Nucl. Instrum. Meth. B. 1994. Vol. 90, No. 1-4. P. 179–182.
Shul'ga N. F., Greenenko A. A. Phys. Lett. B. 1995. Vol. 353, No. 2. P. 373–377.



E=10 TeV

$$\langle \psi^2 \rangle = \frac{lL}{R^2} \leq \psi_c^2$$





Scandale W., Vomiero A., Baricordi S. et al. High-efficiency deflection of high-energy protons through axial channeling in a bent crystal. Phys. Rev. Lett. 2008. Vol. 101, No. 16. P. 164801.
Scandale W., Vomiero A., Bagli E. et al. High-efficiency deflection of high-energy negative particles through axial channeling in a bent crystal. Phys. Lett. B. 2009. Vol. 680, No. 4. P. 301–304.

Crystal Bending

Marco Romagnoni et al. Bent Crystal Design and Characterization for High-Energy Physics Experiments. Crystals 2022, 12, 1263. https:// doi.org/10.3390/cryst12091263

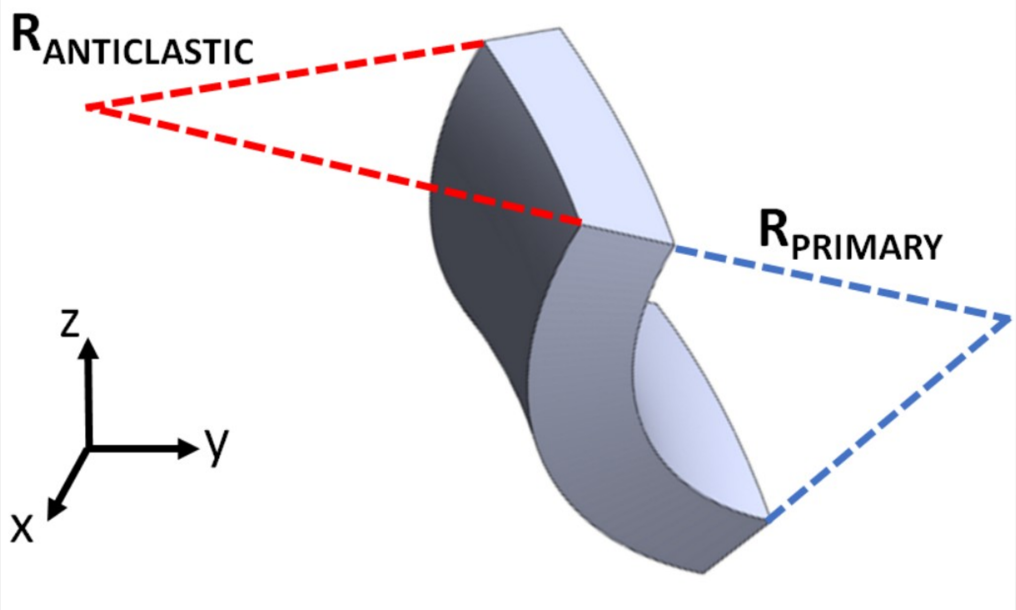


Figure 8. sketch of anticlastic bending state.

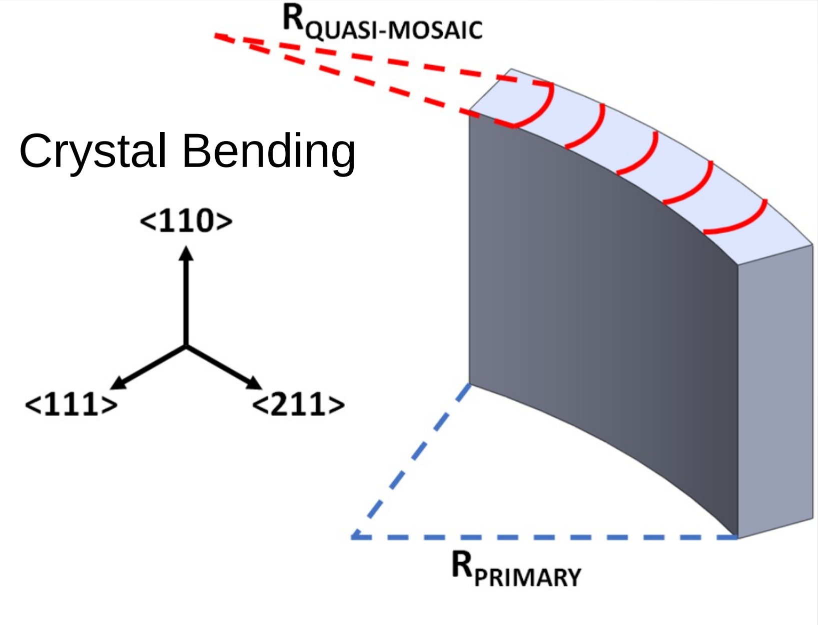
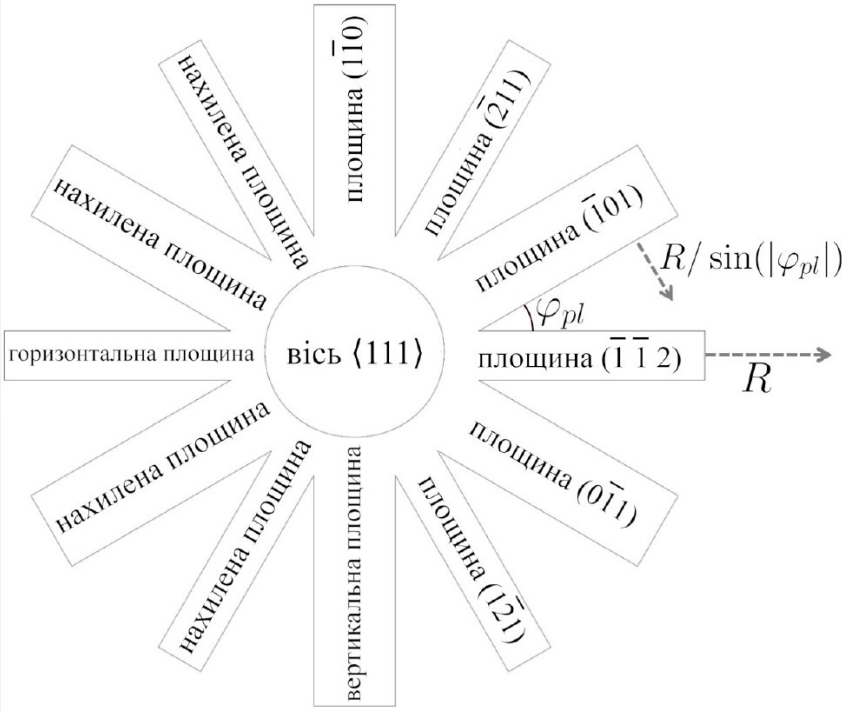


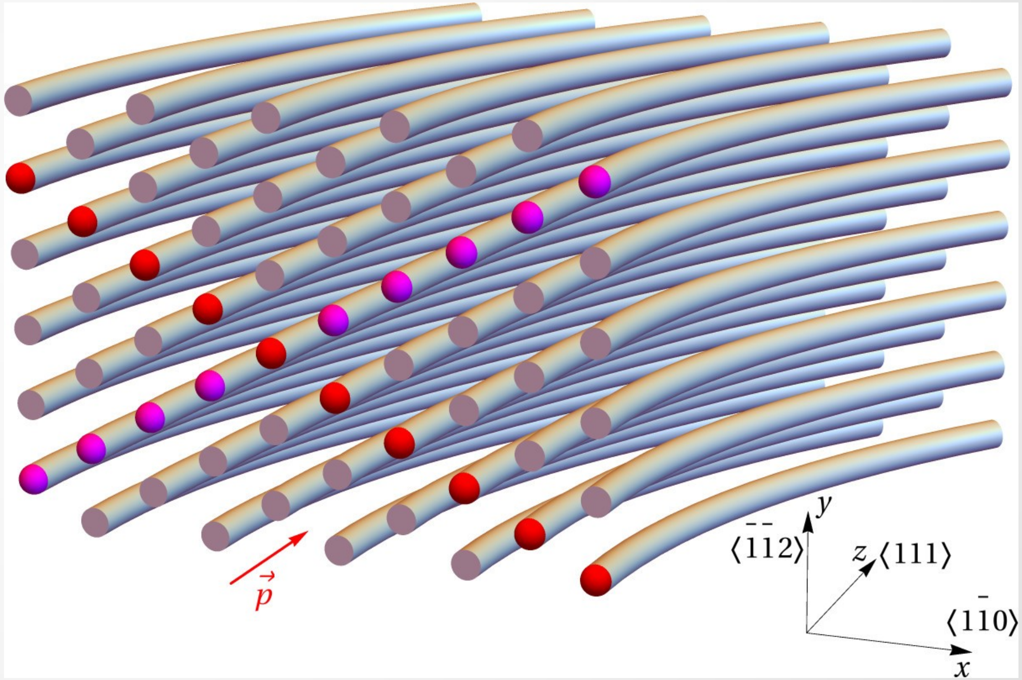
Figure 9. sketch of quasi-mosaic bending state.

Stochastic deflection

$$\alpha_{st} = \frac{2R\psi_c^2}{l_0}$$

$$R_{cr} = \frac{E \sin|\varphi_{pl}|}{\max\left(\left|\frac{\partial U_{pl}(\zeta)}{\partial \zeta}\right|\right)}$$

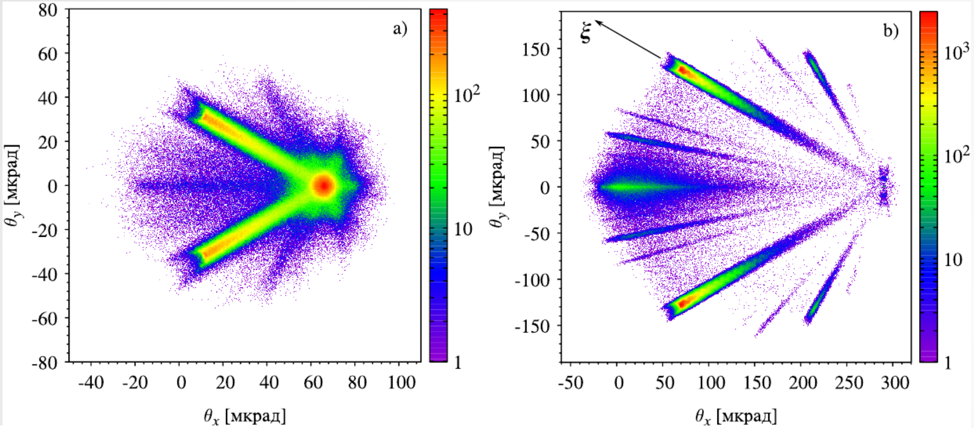


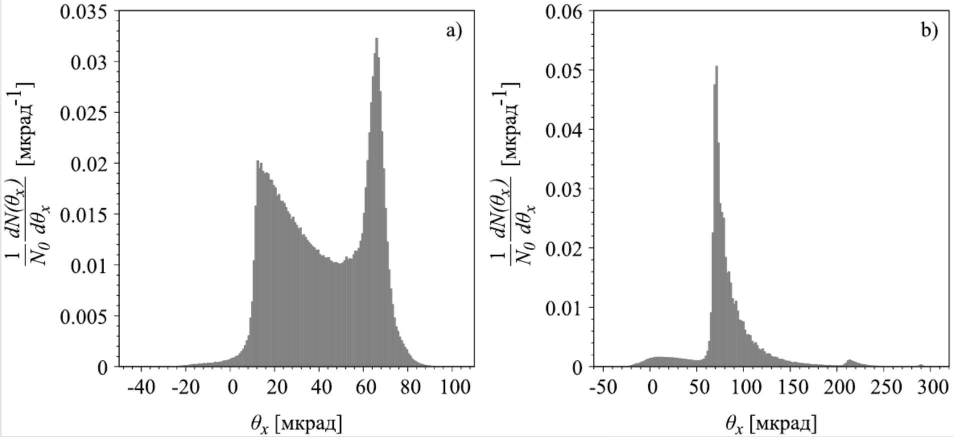


Changing the shape of the beam

R=30,3 m　　　　R=6,9 m

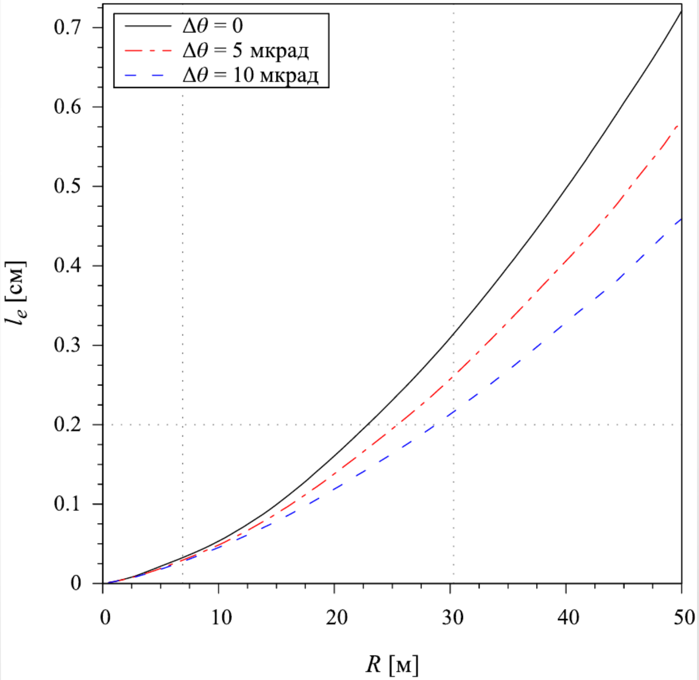
p, E=400 GeV, Si <111>, L=2 mm





$$|\varphi_{pl}| = \pi/6 \rightarrow R_{cr} \approx 35 \text{ см}$$

$$\frac{dN}{dl} = -CN \rightarrow N_{pl}(l) = N_0\left(1 - e^{-l/l_e}\right)$$

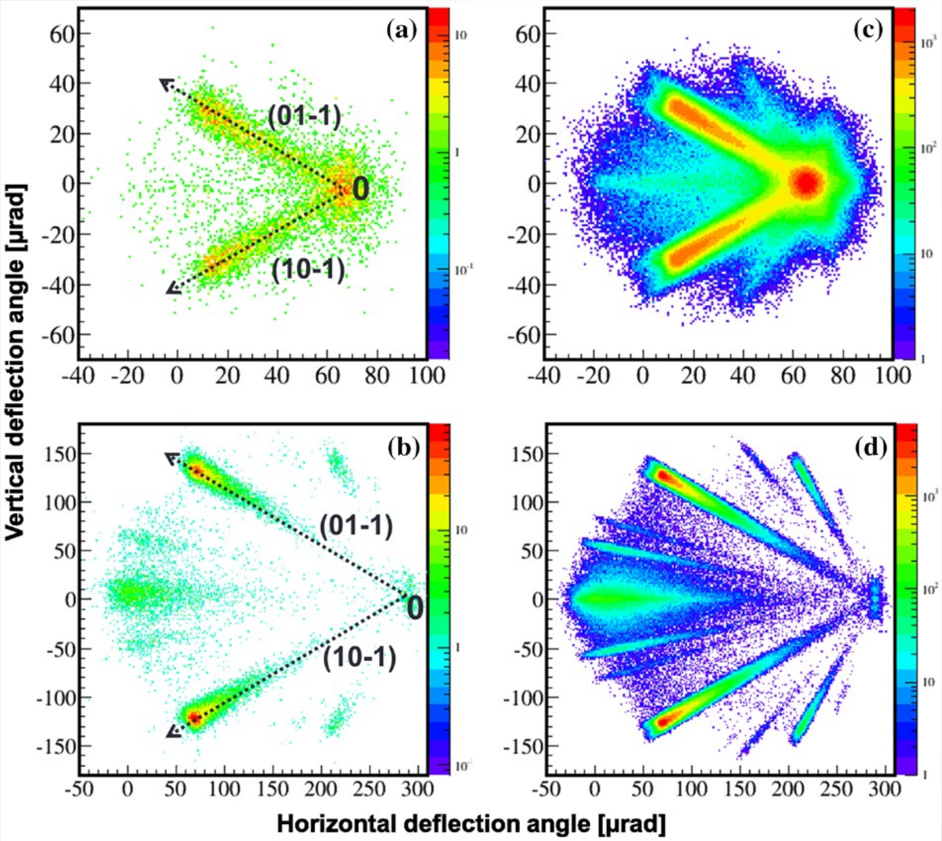


Changing the shape of the beam

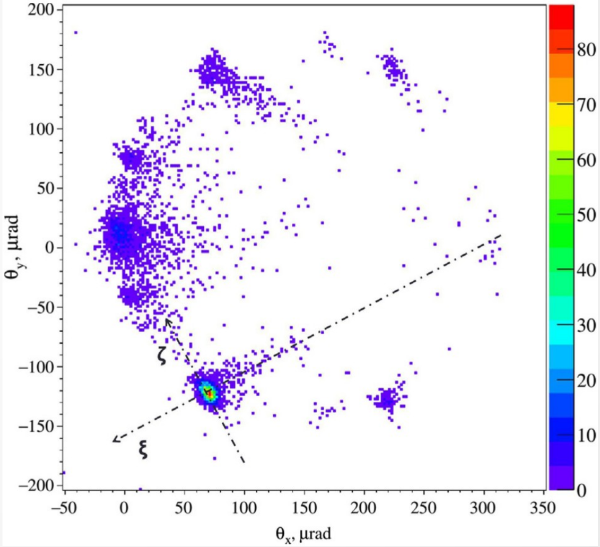
p, *E*=400 GeV, Si <111>, *L*=2 mm

R=30,3 m

R=6,9 m



p, *E*=400 GeV, Si <111>, *L*=2 mm, R=6,9 m, $\theta_{x,in}$=-8 μrad, $\theta_{y,in}$=-4 μrad

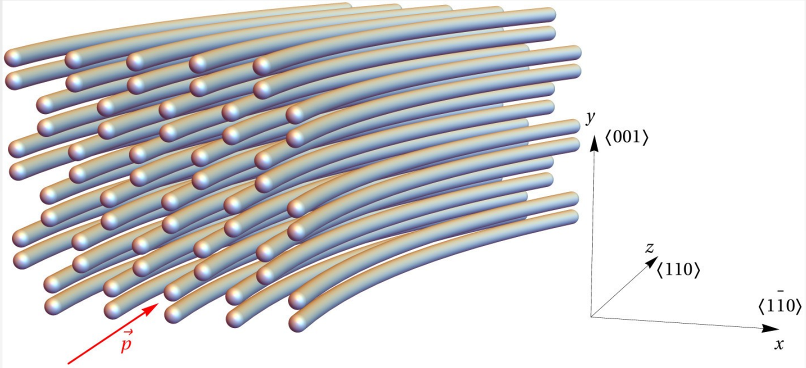


Bandiera L., Mazzolari A., Bagli E. et al. (Kirillin I. V.). Relaxation of axially confined 400 GeV/c protons to planar channeling in a bent crystal. Eur. Phys. J. C. 2016. Vol. 76. P. 80 (1–6).

Bandiera L., Kirillin I. V., Bagli E. et al. Splitting of a high-energy positively-charged particle beam with a bent crystal. Nucl. Instr. Meth. Phys. Res. B. 2017. Vol. 402. P. 296–299.

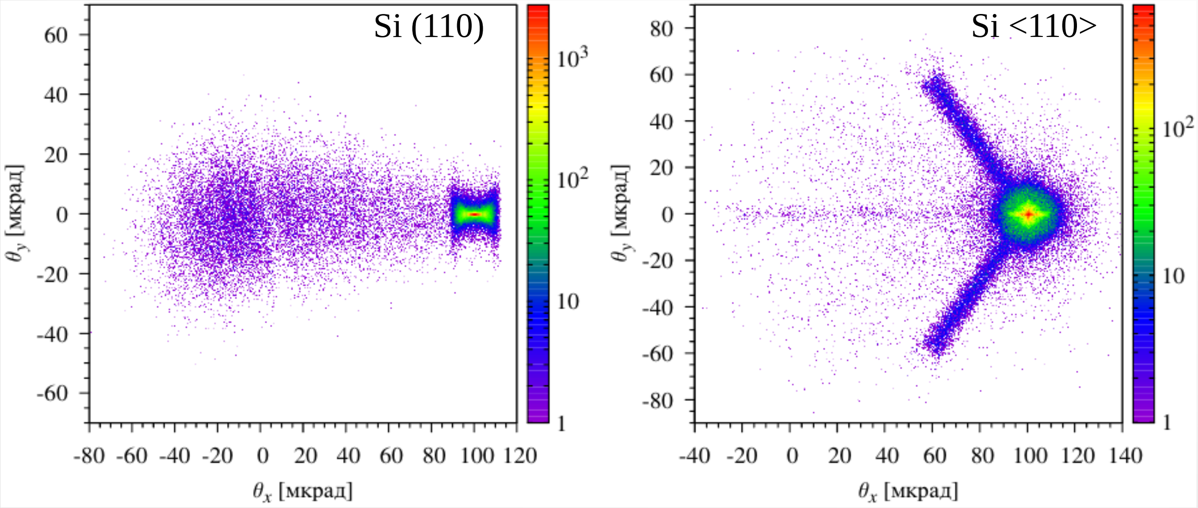
Probability of close collisions

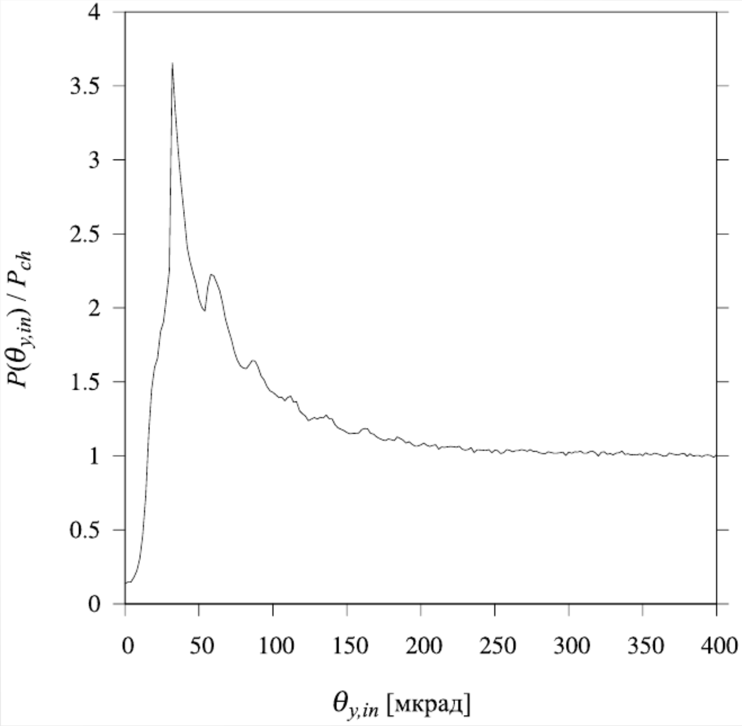
p, *E*=270 GeV,
Si <110>, (110),
L = 5 mm,
R = 5 m



$$w_a = \frac{4\pi r_T^2}{a_x a_y} = 4\sqrt{2}\pi r_T^2/a^2 \approx 3.39 * 10^{-3}$$

$$w_p = \frac{4 r_T}{a_x} = 4\sqrt{2} r_T/a \approx 78.12 * 10^{-3}$$

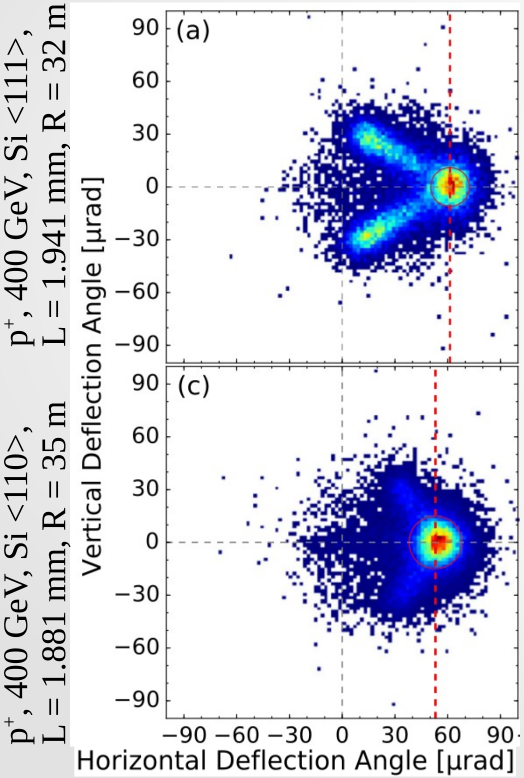




Chesnokov Yu.A., Kirillin I.V., Scandale W. et al. Phys. Lett. B. 2014. Vol. 731. P. 118–121.

Probability of close collisions

Scandale W., Arduini G., Butcher M. et al. Phys. Lett. B. 2016. Vol. 760. P. 826–831.
Scandale W., Andrisani F., Arduini G. et al. Eur. Phys. J. C. 2018. Vol. 78, No. 6. P. 505.



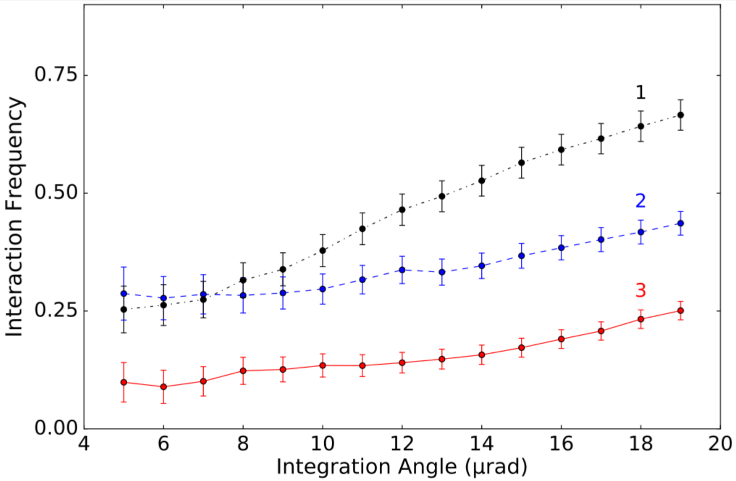
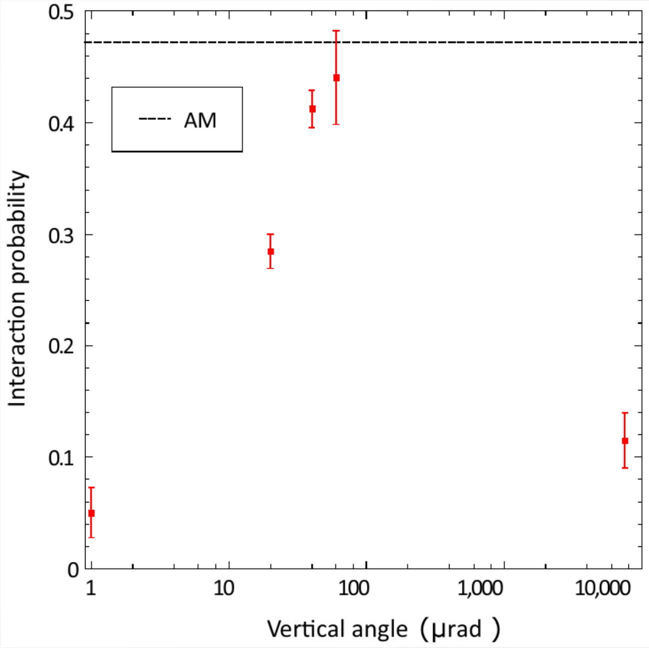
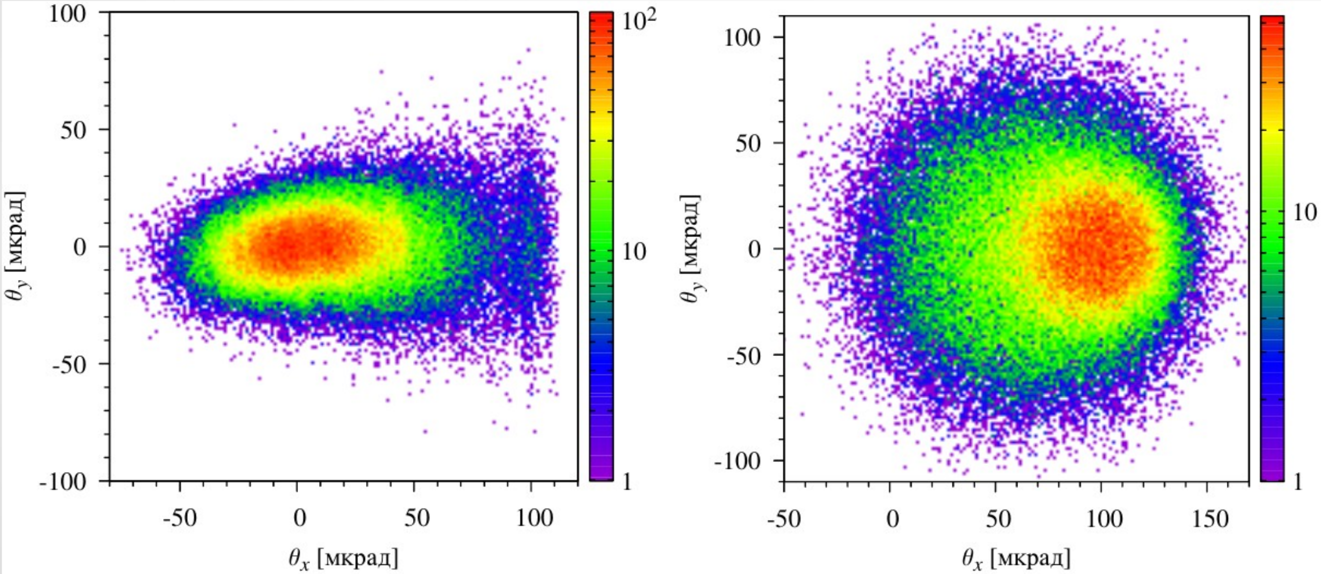


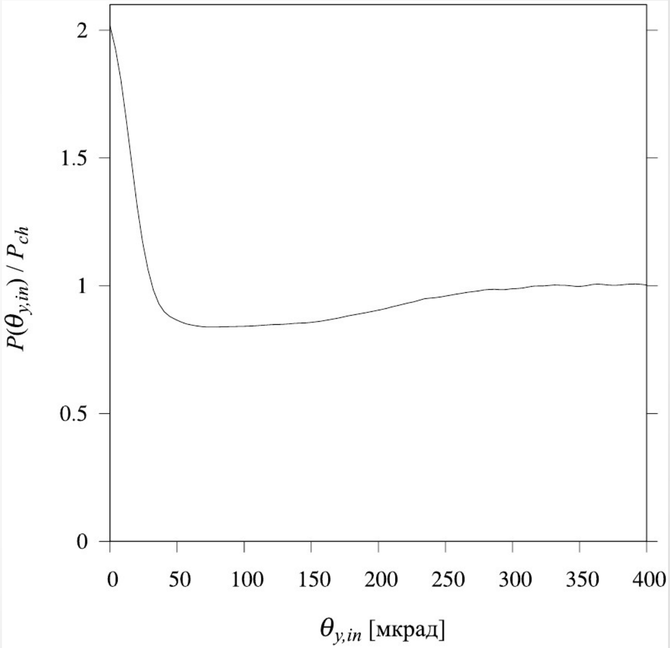
Fig. 5. Measured inelastic nuclear interaction (INI) frequency of 400 GeV/c protons interacting with the ⟨111⟩ and ⟨110⟩ crystals as a function of the angular region around the (110) planar channeling (black dash-dotted line, 1), the ⟨111⟩ axial channeling (blue dashed line, 2) and ⟨110⟩ (red continuous line, 3) orientations. The values are normalized to the INI frequencies for the amorphous crystal orientation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



Probability of close collisions

π^-, E = 270 ГэВ, Si <110>, L = 5 mm, R = 5 m





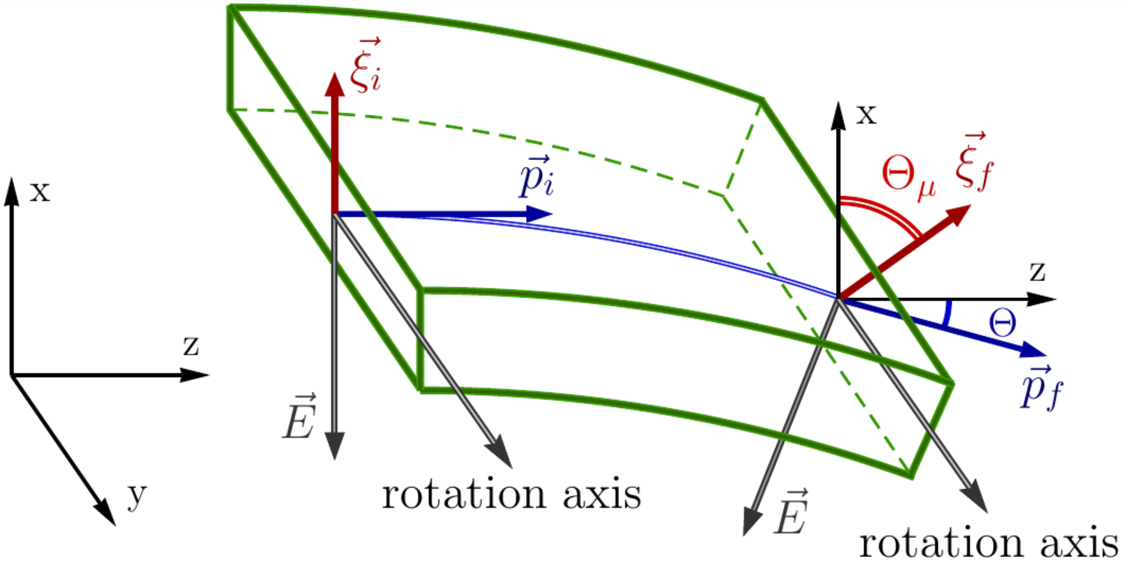
Kirillin I.V., Shul’ga N.F. Nucl. Instr. Meth. Phys. Res. B. 2015. Vol. 355. P. 49–52.

Feasibility of measuring the magnetic dipole moments of the charm baryons at the LHC using bent crystals

Fomin A.S., Korchin A.Yu., Stocchi A. et al. (S.P. Fomin, I.V. Kirillin, N.F. Shul'ga) J. High Energy Phys. 2017. Vol. 2017. № 8. P. 120 (1–26).

$$\Theta_\mu = \gamma\left(\frac{g}{2} - 1 - \frac{g}{2\gamma^2} + \frac{1}{\gamma}\right)\Theta \approx \gamma\left(\frac{g}{2} - 1\right)\Theta$$

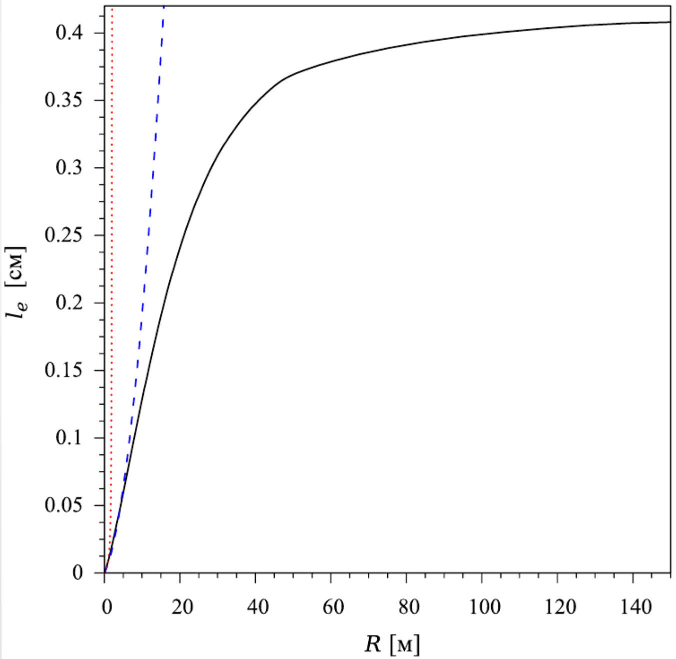
(V.G. Baryshevsky, 1979; V.L. Lyuboshits, 1980)

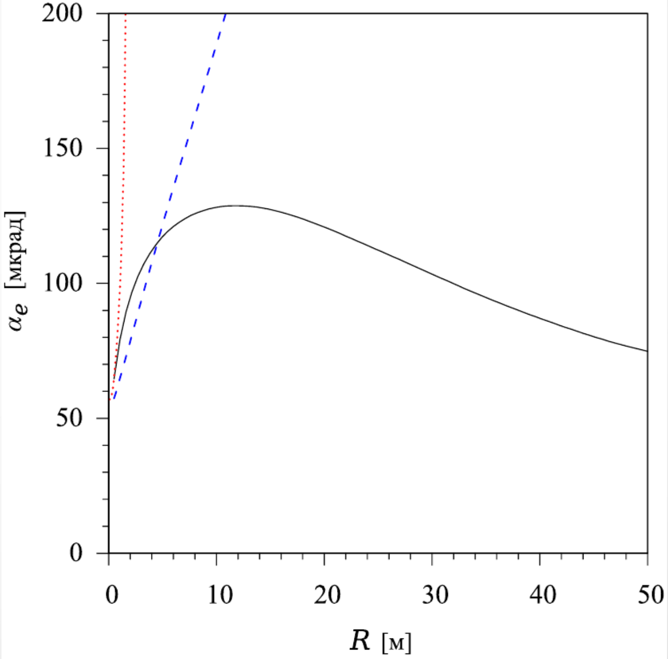


Optimal radius of curvature (stochastic deflection)

$$\langle\psi^2\rangle = \frac{lL}{R^2} \le \psi_c^2 \qquad \overline{\Psi_{inc}}^2 = \xi L \qquad L_{st} = \frac{\psi_m^2}{l/R^2 + \xi} \qquad \alpha_{st} = \frac{L_{st}}{R} = \frac{\psi_m^2}{l/R + \xi R}$$

π^-, E=150 GeV, Si <110>





Kirillin I.V., Shul'ga N.F., Bandiera L. et al. Eur. Phys. J. C. 2017. Vol. 77. P. 117 (1–7).

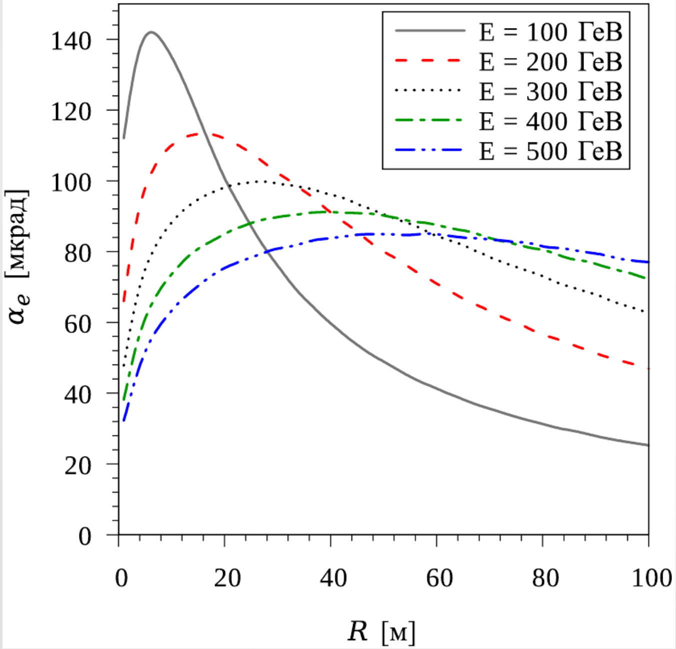
Optimal radius of curvature (stochastic deflection)

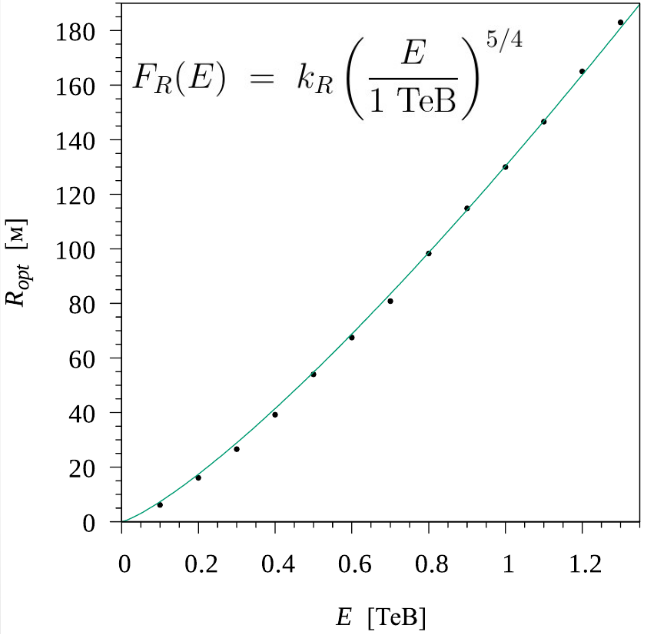
$$\overline{\psi_{inc}^2} = \zeta L/E^2 \Rightarrow \alpha_{st} = \frac{\psi_m^2}{l/R + \zeta R/E^2} \Rightarrow R_{opt} = E\sqrt{l/\zeta}$$

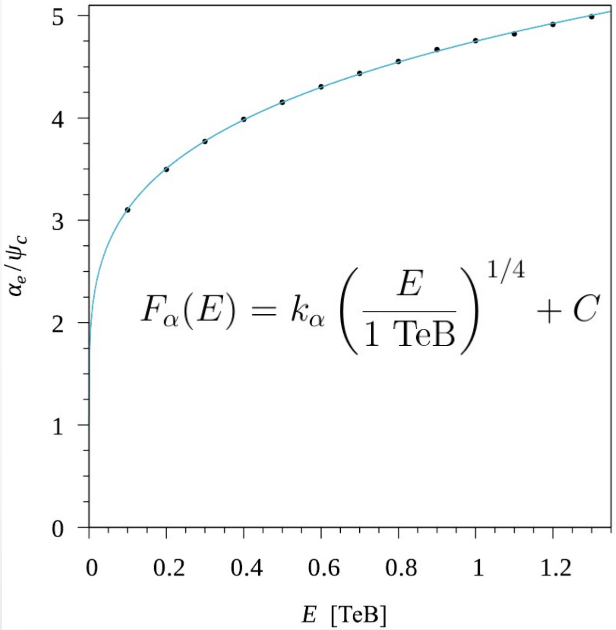
$$l \approx \frac{1}{4nda}\sqrt{\frac{E}{U_0}} \Rightarrow R_{opt} \propto E^{5/4}$$

$$\psi_m \approx 1,5\psi_c \propto E^{-1/2} \Rightarrow \max(\alpha_{st}) = \frac{\psi_m^2}{l/R_{opt} + \zeta R_{opt}/E^2} \propto E^{-1/4}$$

π^-, Si <110>







$k_R \approx 130$ м

$k_\alpha \approx 3,75$ і $C \approx 1$

Optimal radius of curvature (planar channeling)

$$\frac{d^2x}{dt^2} = -\frac{c^2}{E}\frac{dU_{\text{eff}}(x)}{dx}$$

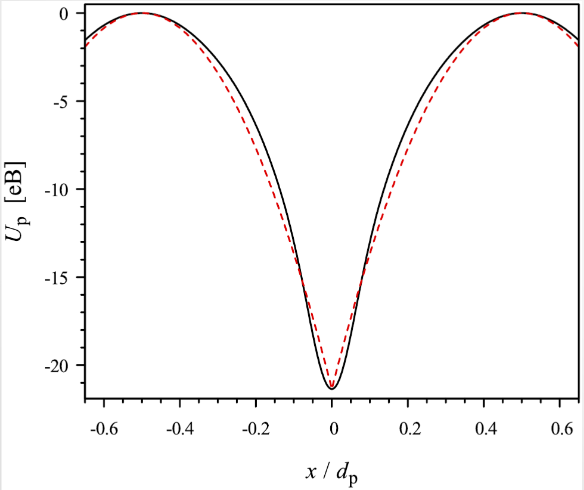
$$U_{\text{eff}}(x) = U_{\text{p}}(x) + Ex/R$$

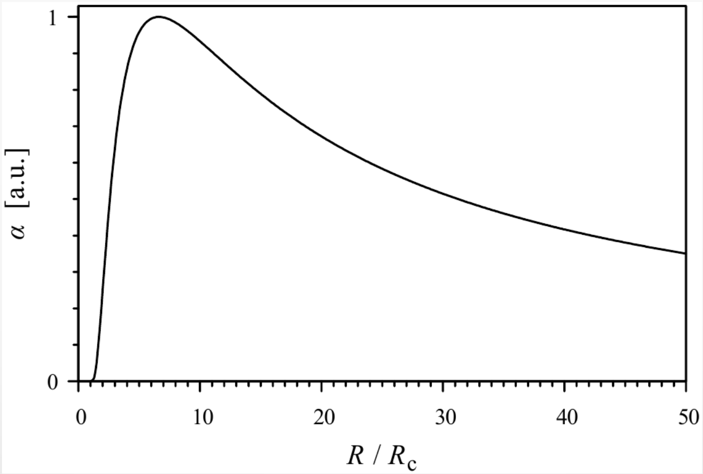
$$U_{\text{p}}(x) = -\frac{U_0}{d_{\text{p}}^2}\left((2x+d_{\text{p}})^2 H(-x(d_{\text{p}}+x)) + \right.$$
$$\left. + (2x-d_{\text{p}})^2 H(x(d_{\text{p}}-x))\right)$$

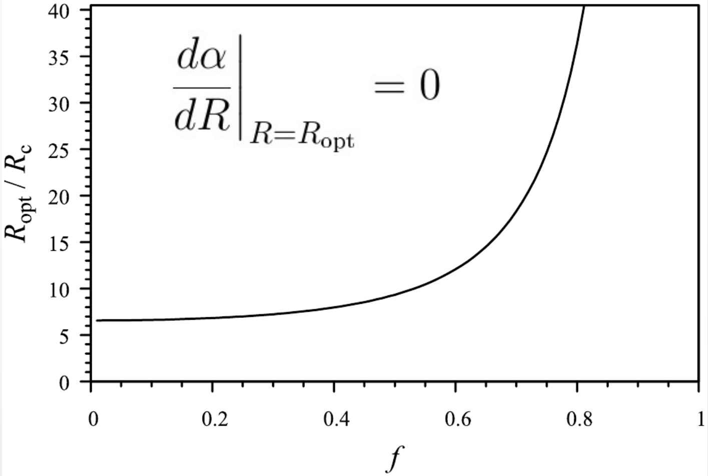
$$R \;>\; R_{\text{c}}$$

$$\frac{x_{\text{pos}} - x_{\text{neg}}}{d_{\text{p}}} = 1 - \sqrt{\frac{R_{\text{c}}}{R}}$$

$$\alpha = \frac{l}{R} = \frac{\theta_{\text{c}}^2}{2\xi_{\text{p}}^2 R\left(\text{erf}^{-1}\left(\frac{f}{1-\sqrt{\frac{R_{\text{c}}}{R}}}\right)\right)^2}$$



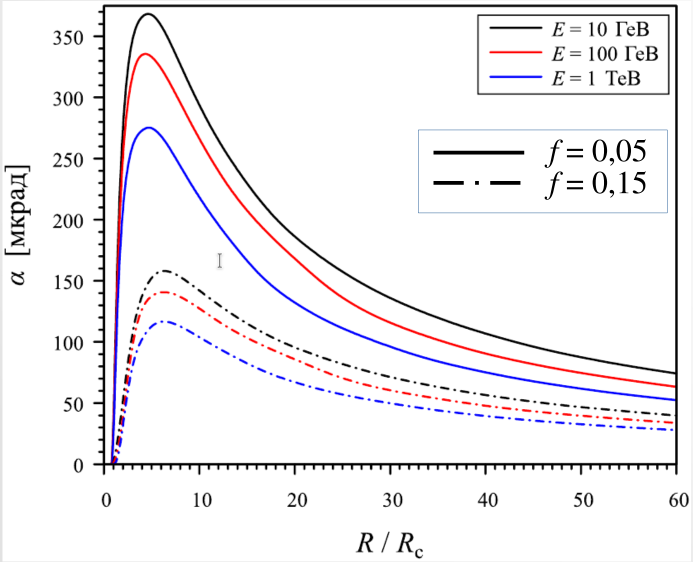


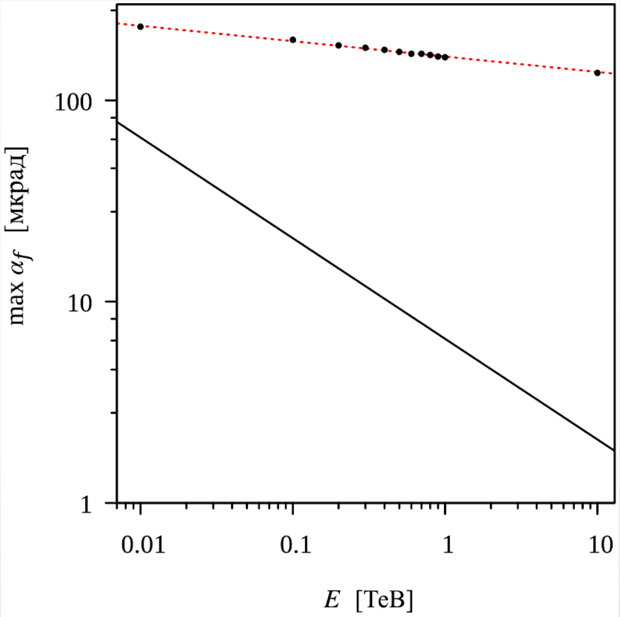


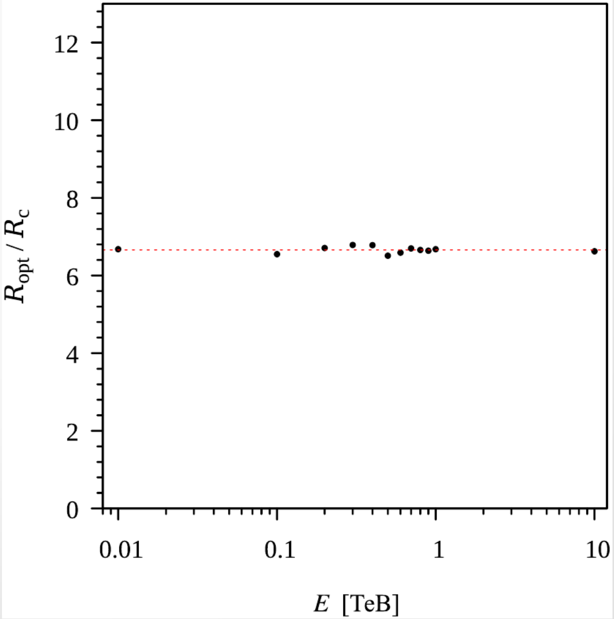
Optimal radius of curvature (planar channeling)

$$U_{\mathrm{str}}(\rho) = -\frac{8\pi^2\hbar^2}{m_{\mathrm{e}}d}\sum_{k=1}^{4}\frac{\alpha_k}{\beta_k + B}e^{-\frac{4\pi^2\rho^2}{\beta_k+B}} \Rightarrow U_{\mathrm{pl}}(x) = -\frac{4\pi^{\frac{3}{2}}\hbar^2}{m_{\mathrm{e}}dd_{\mathrm{s}}}\sum_{k=1}^{4}\frac{\alpha_k}{\sqrt{\beta_k + B}}e^{-\frac{4\pi^2x^2}{\beta_k+B}}$$

$$U_{\mathrm{p}}(x) = \sum_{n=-\infty}^{\infty}U_{\mathrm{pl}}(x - x_n) \Rightarrow U_{\mathrm{p}}(x) = -\frac{2\pi\hbar^2}{m_{\mathrm{e}}dd_{\mathrm{s}}d_{\mathrm{p}}}\sum_{k=1}^{4}\alpha_k\theta_3\left(\pi\frac{x}{d_p}, e^{-\frac{\beta_k+B}{4d_{\mathrm{p}}^2}}\right)$$







W. Scandale et al. Observation of channeling for 6500 GeV/c protons in the crystal assisted collimation setup for LHC. Phys. Lett. B 758 (2016) 129

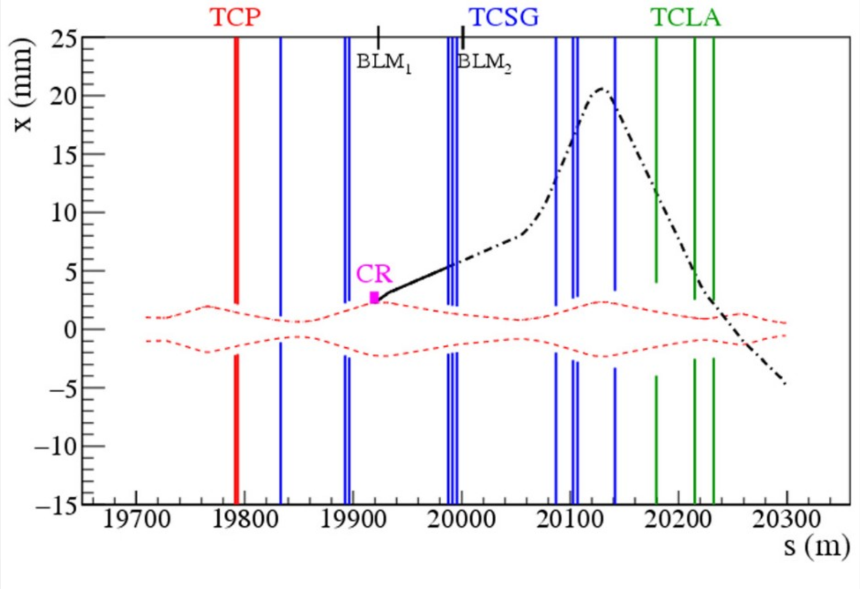


Fig. 1. (Color online.) The horizontal projection of the trajectory of a halo particle deflected by the crystal due to channeling at the bend angle $\alpha = 65$ µrad (solid line up to the first horizontal $TCSG_1$ and then by dot-dashed line to show the trajectory propagation in the case without the collimators): (a) for the beam injection with

Marco D'Andrea et al. Characterization of bent crystals for beam collimation with 6.8 TeV proton beams at the LHC. NIM A 1060 (2024) 169062

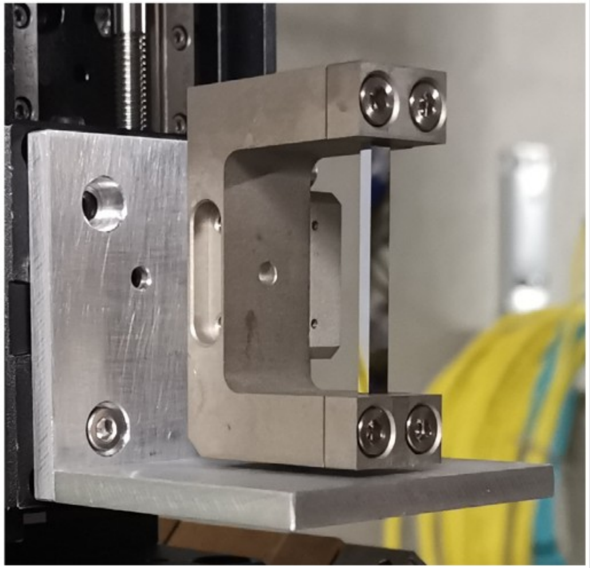
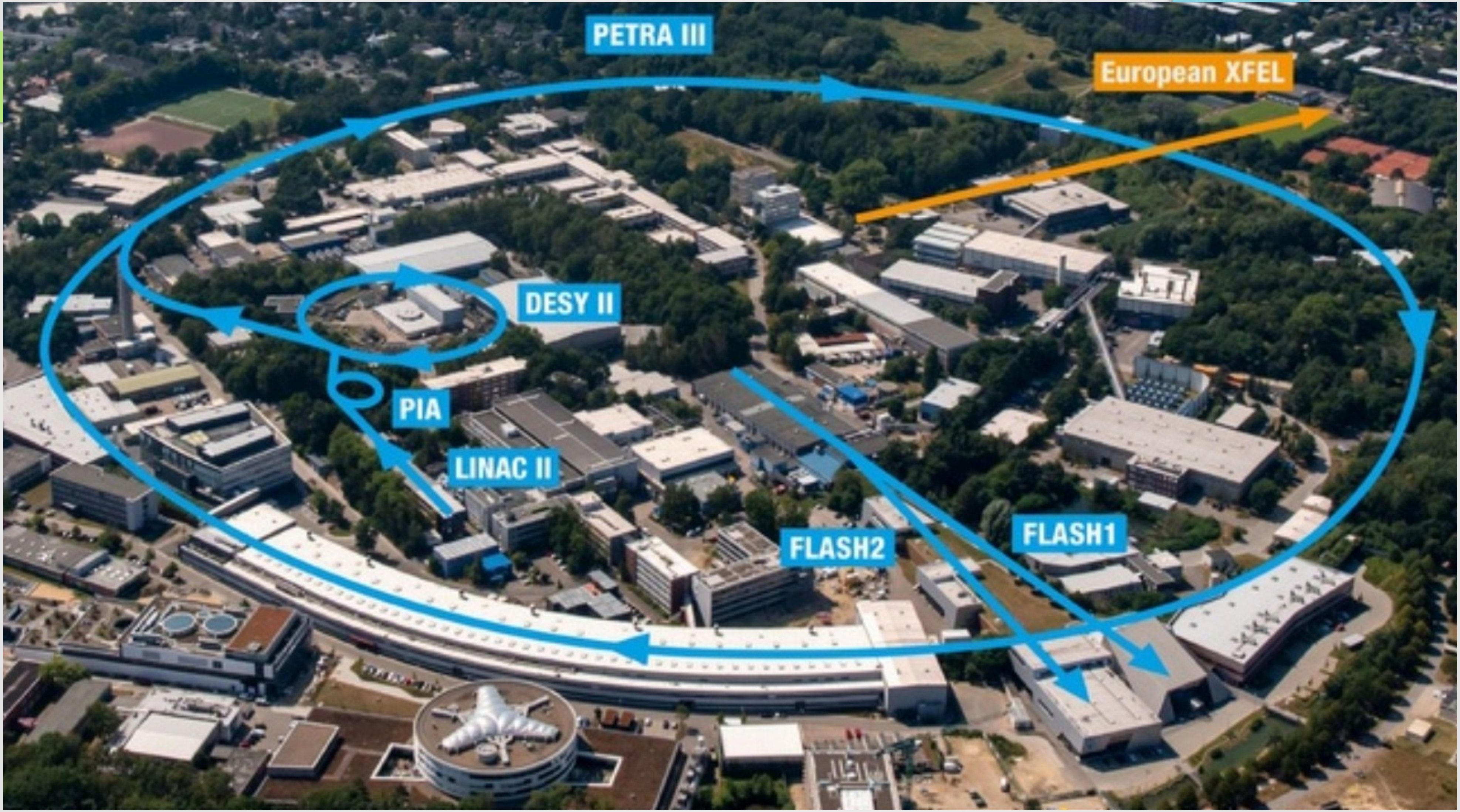


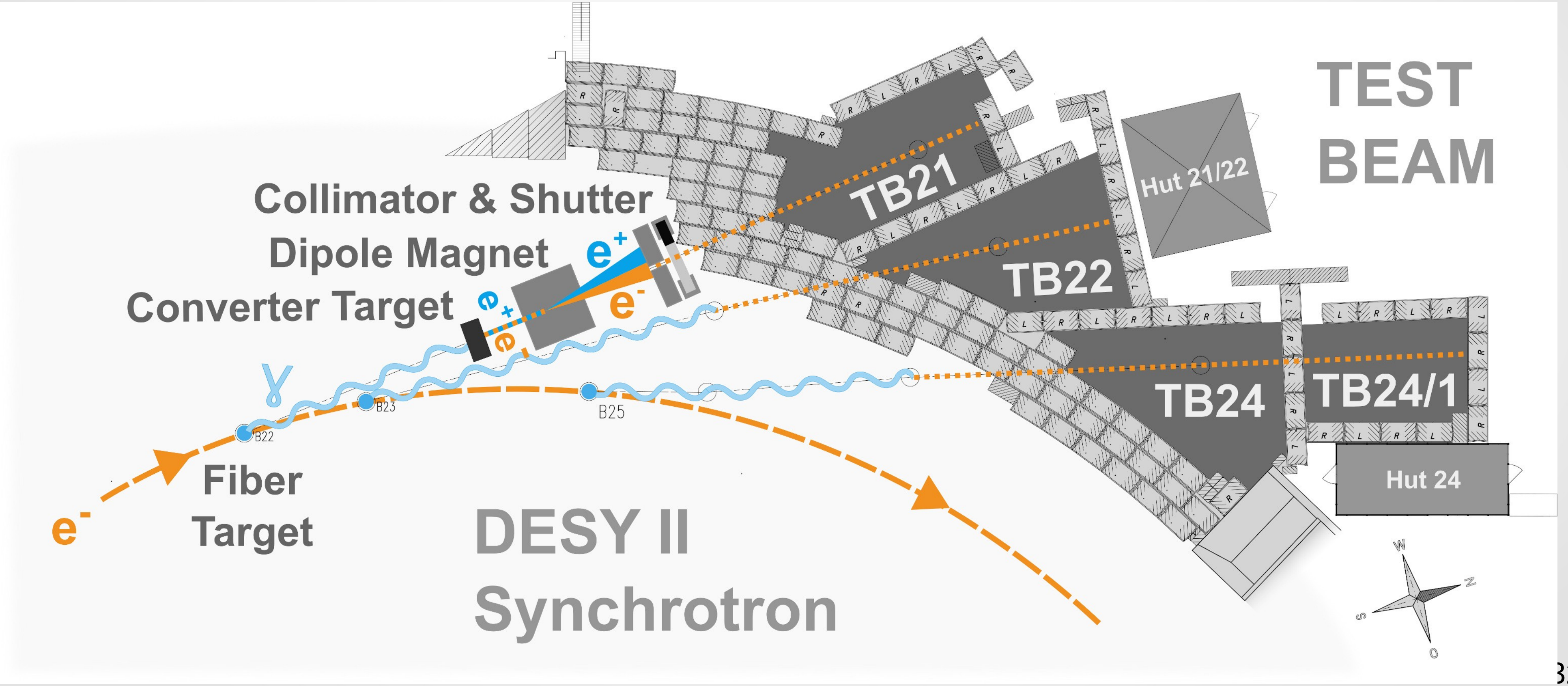
Fig. 2. Picture of a Si crystal clamped by its metal holder.

TCP -- primary collimators
TCSG -- secondary collimators

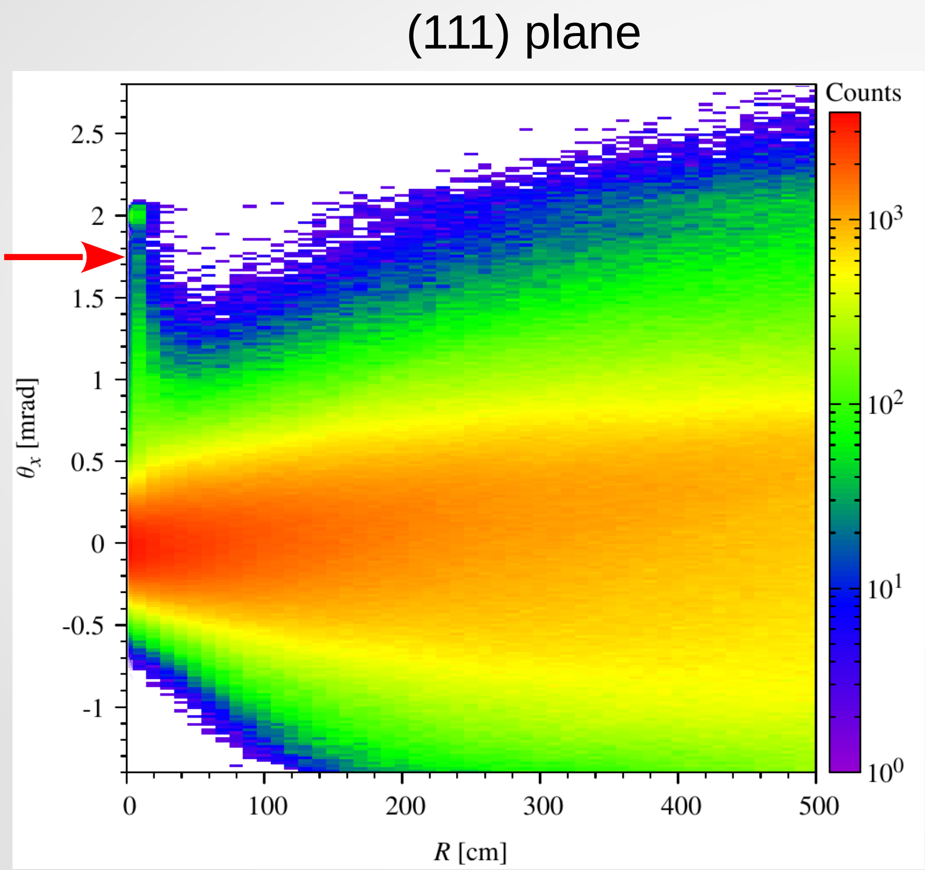
TCLA -- shower-absorber collimators made from a tungsten heavy alloy

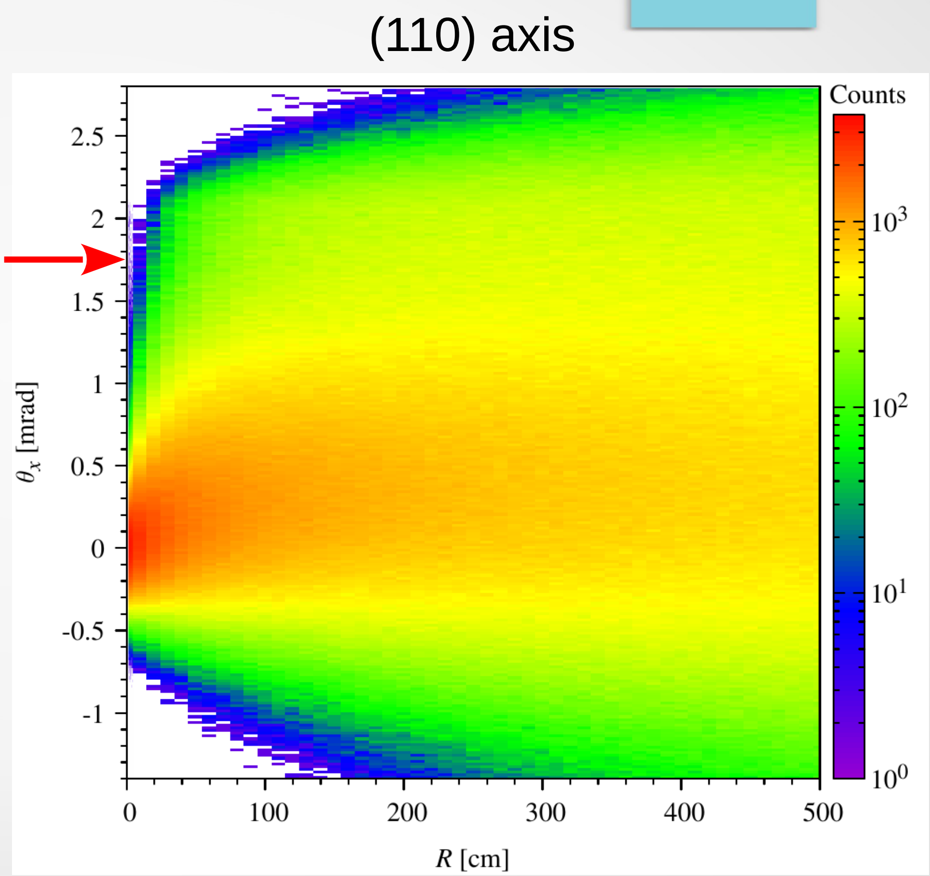

PETRA III
European XFEL
DESY II
PIA
LINAC II
FLASH2
FLASH1

Generation of the DESY Test Beams



Dependence of deflection angle on R (α = 2 mrad)





Thank you for your attention!