PLASMA WAKEFIELD ACCELERATION AND FOCUSING

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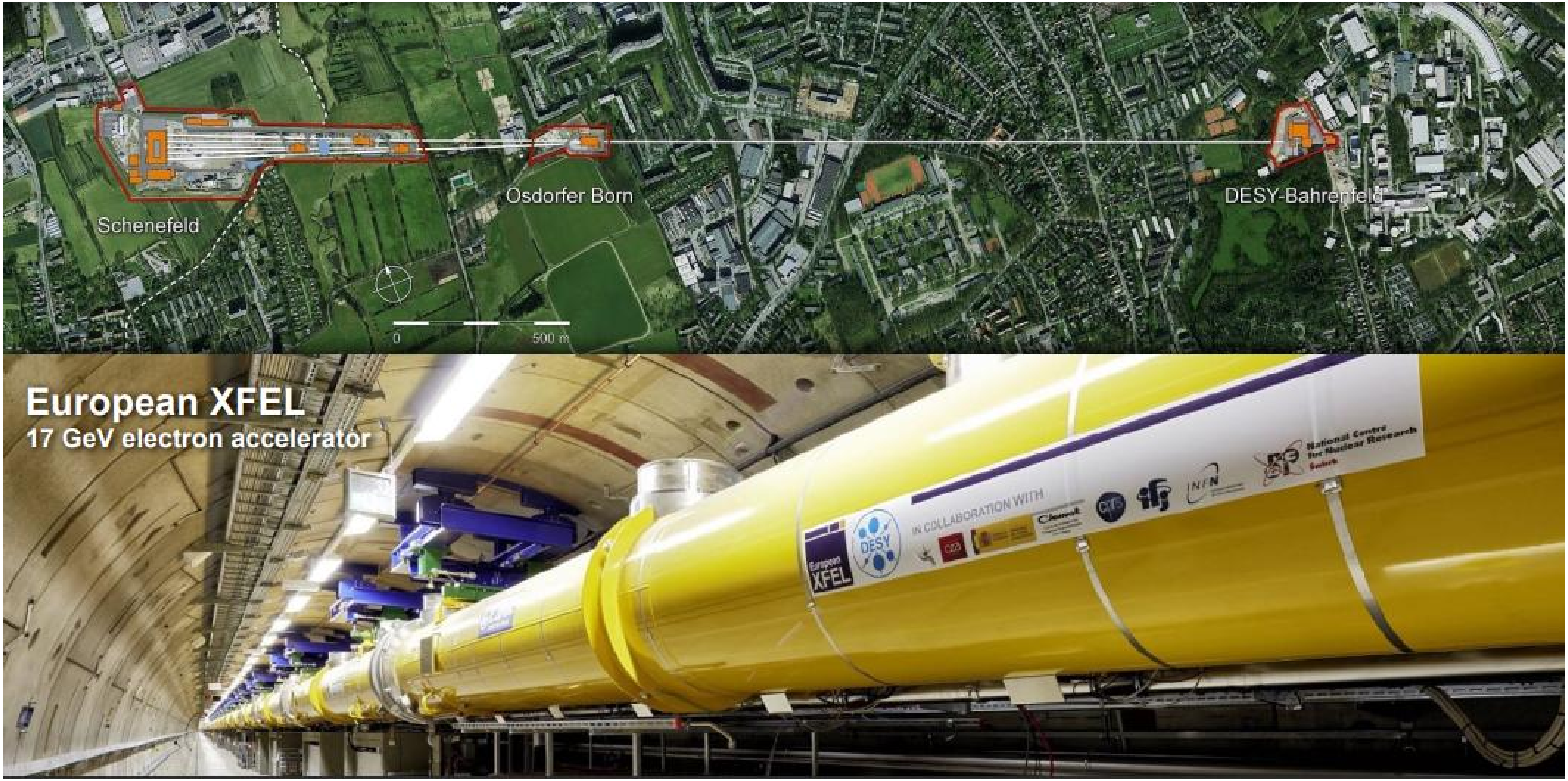
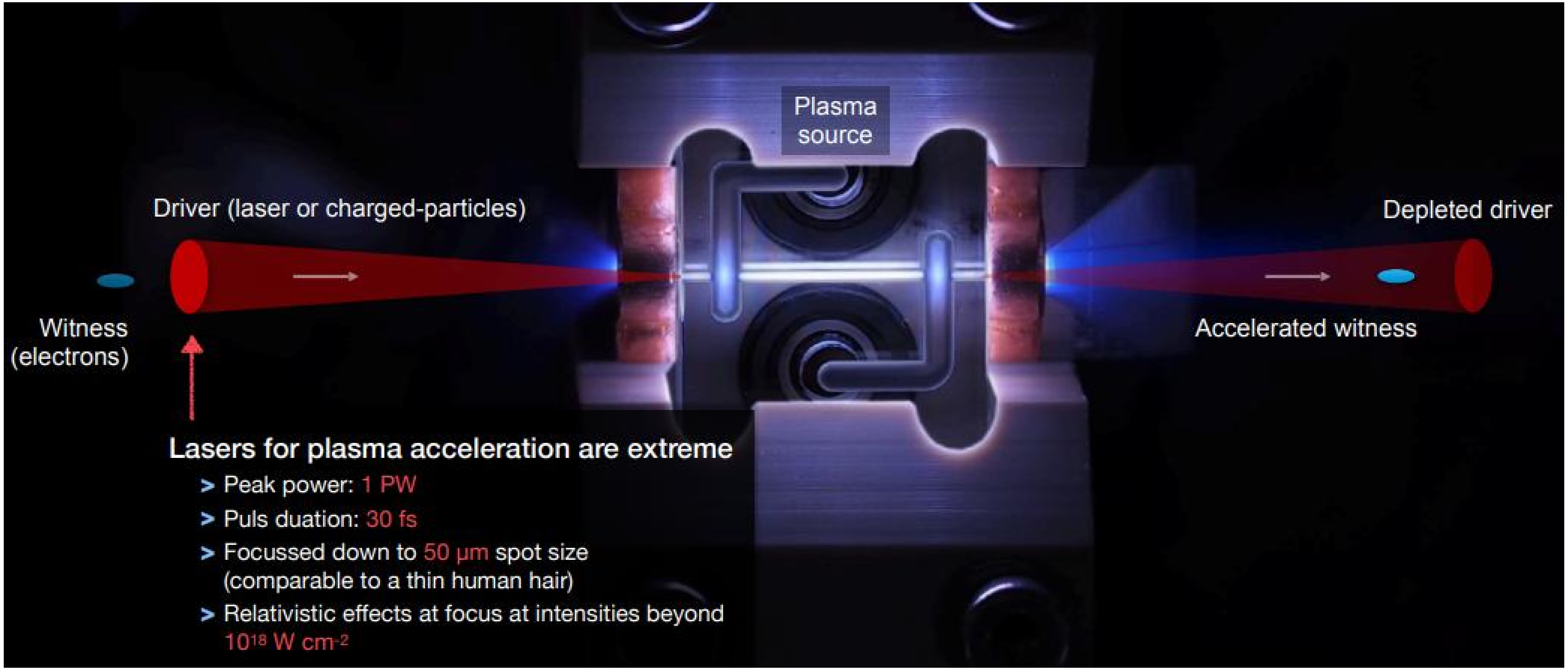
DESY.

Accelerators for research are powerful, **large-scale machines**



from Jens Osterhoff lecture, DESY Winter School (2023)



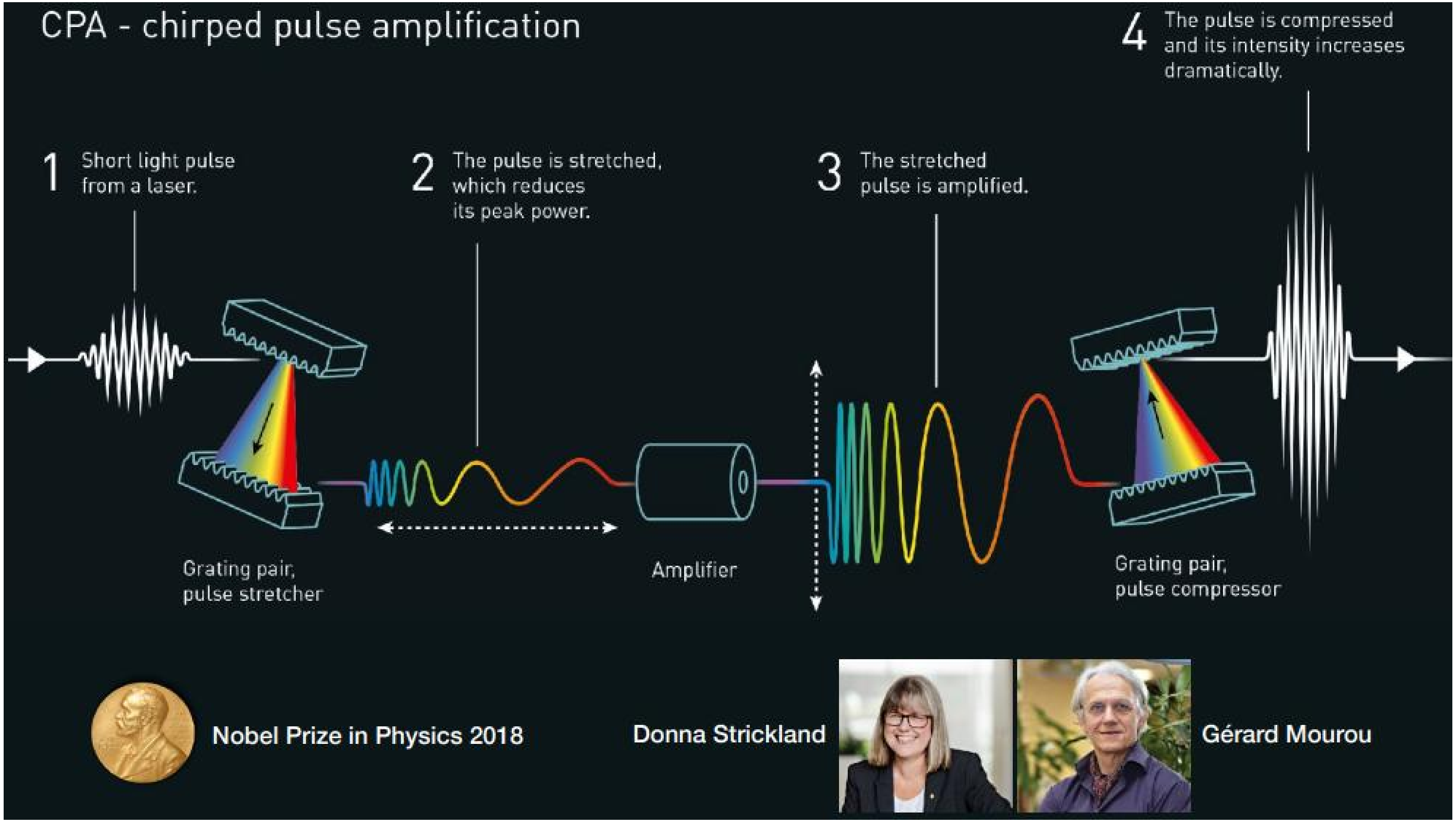
Why laser driven? → ultra-compact laser driver technology

Problems: low repetition rates, low average power, low wall-plug efficiency

Why beam driven? → high repetition rate and high average power

Problem: requires a relatively large traditional driver accelerator

from Jens Osterhoff lecture, DESY Winter School (2023)



from Jens Osterhoff lecture, DESY Winter School (2023)

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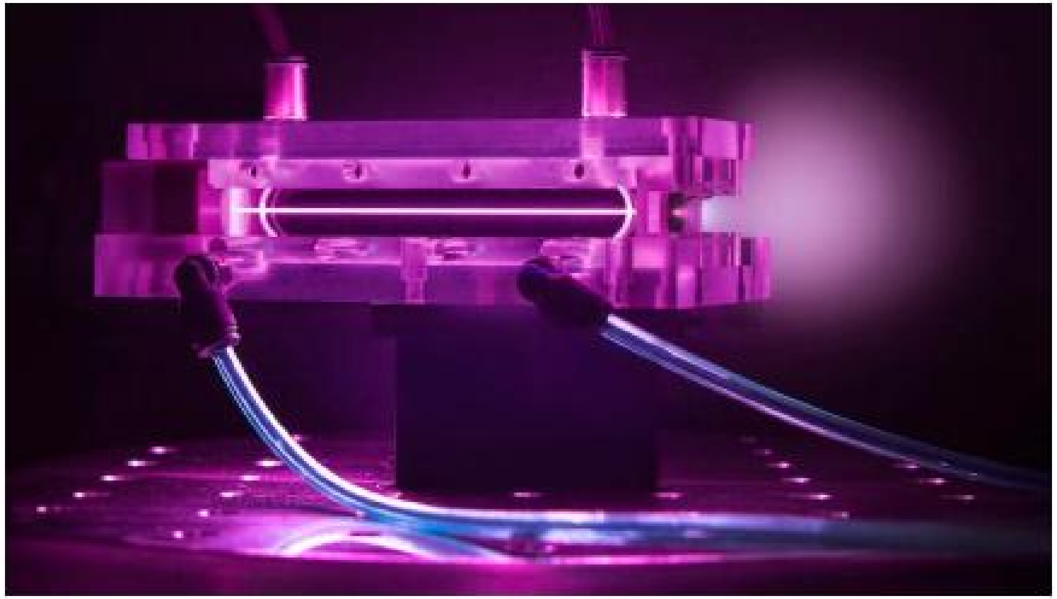
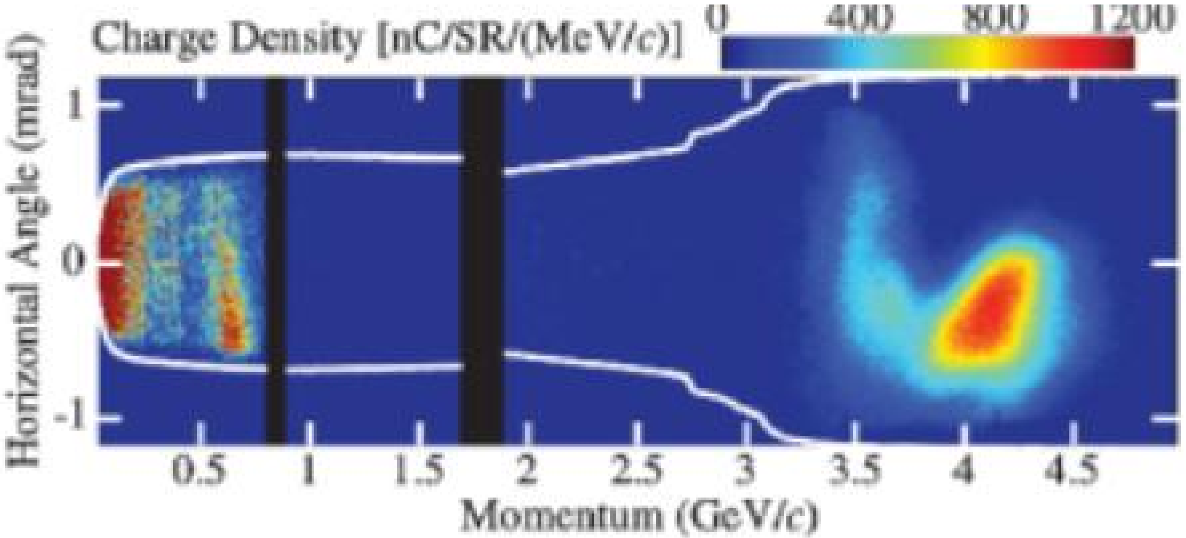
Laser plasma acceleration of up to 8 GeV electrons



W. Leemans *et al.*, PRL **113**, 245002 (2014)



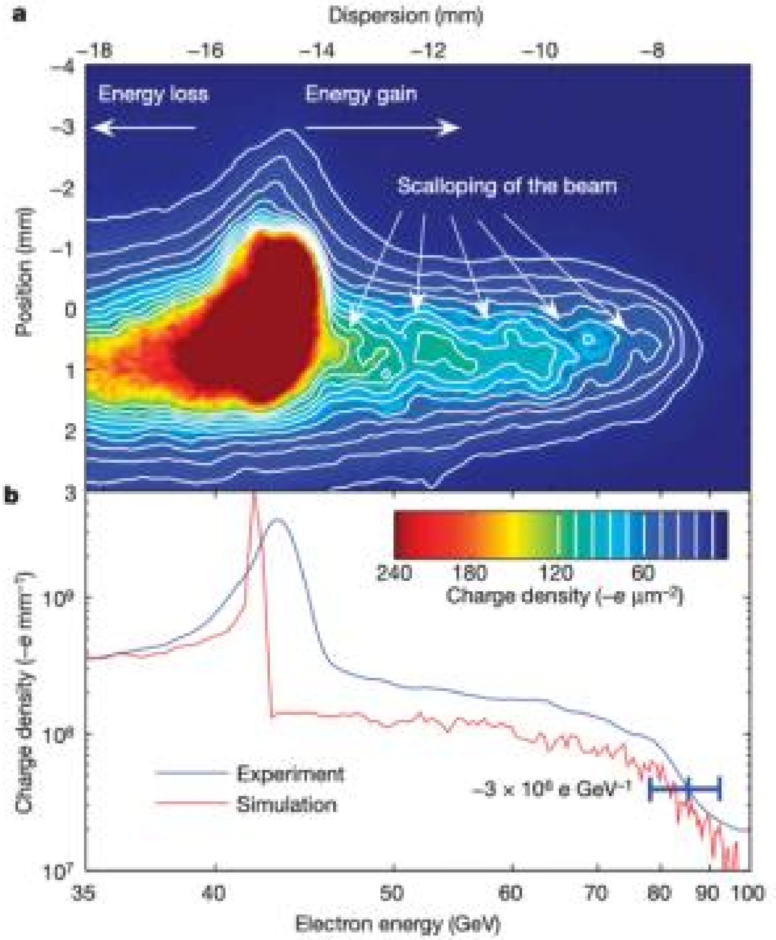
W.Leemans et al., PRL 113, 245002 (2014)

Beam-driven plasma acceleration up to 85 GeV electrons

Plasma accelerator

Energy doubling

85 cm plasma source, 42 GeV driver



I. Blumenfeld *et al.*, Nature **445**, 741 (2007)

Design of a TeV-class LWFA-based linear collider

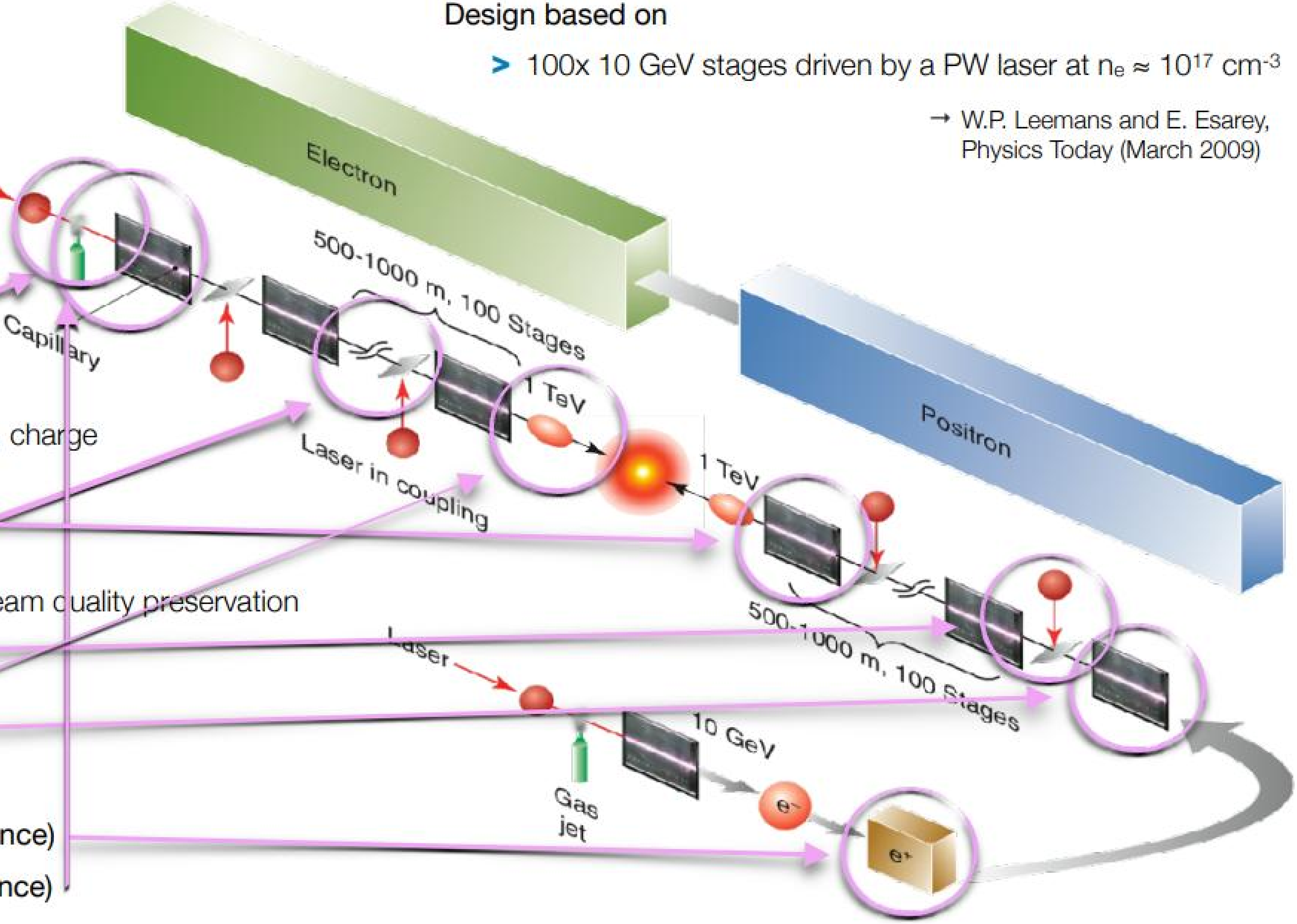
Design based on

- 100x 10 GeV stages driven by a PW laser at $n_e \approx 10^{17}$ cm^{-3}

→ W.P. Leemans and E. Esarey, Physics Today (March 2009)

Challenges / required R&D

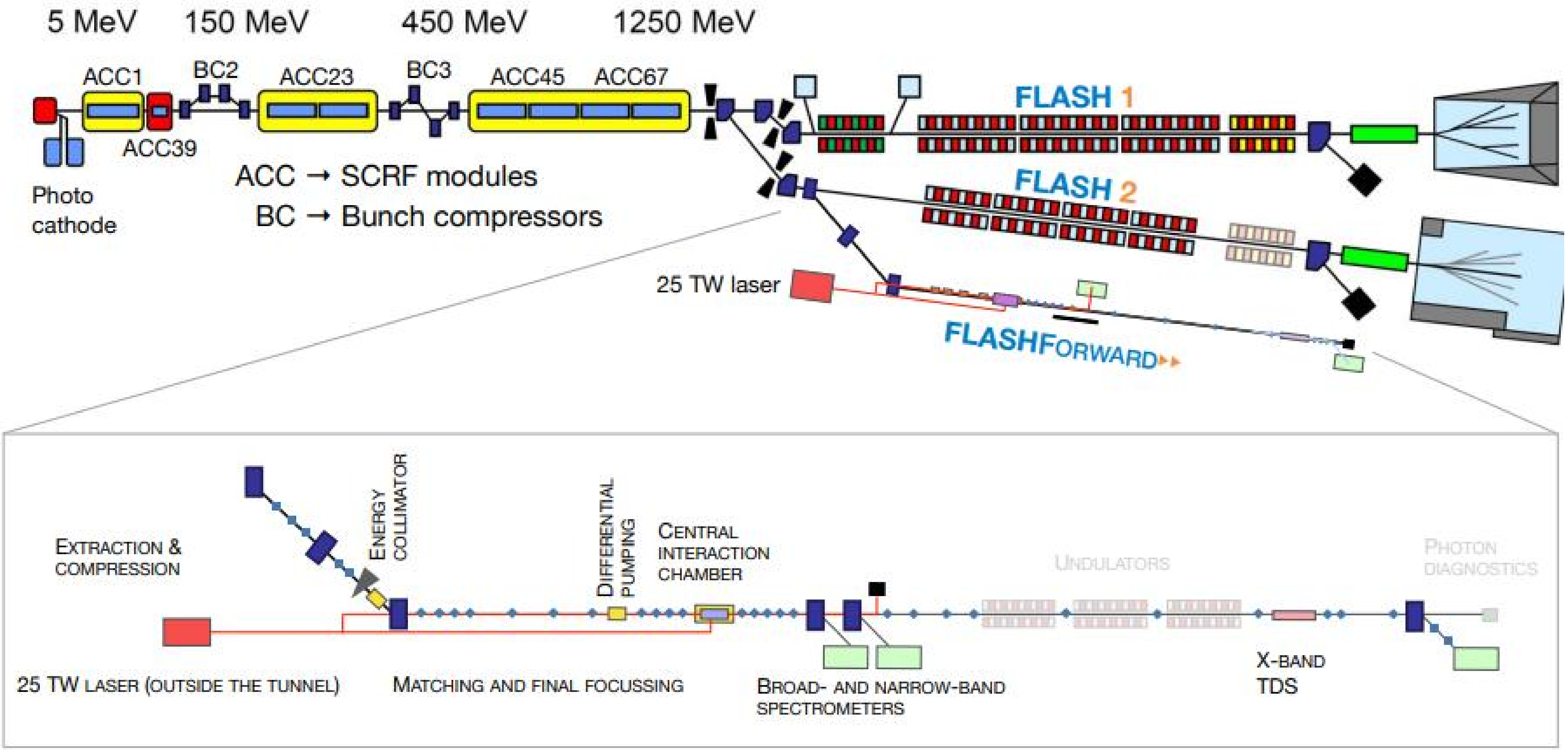
- Laser system
  high peak power,
  high average power + efficiency
- Single acceleration module
  deliver consistent beam quality
  → emittance, energy, energy spread, charge
  efficient energy transfer of driver to witness
- Heat management
- Coupling of two plasma stages
  electron beam extraction and injection → beam quality preservation
- Laser in-coupling
  → accelerator length
- Positron acceleration
- Final focus system
- Positron beam generation (with low emittance)
- Electron beam generation (with low emittance)



from Jens Osterhoff lecture, DESY Winter School (2023)

FLASHFORWARD uses FLASH superconducting accelerator

Plasma accelerator integrated into facility for additional acceleration and benefits from Free-Electron Laser beam



R. D'Arcy *et al.*, Phil. Trans. R. Soc. A **377**, 20180392 (2019)

from Jens Osterhoff lecture, DESY Winter School (2023)

Schemes for Simultaneous Large Transformer Ratio, High Efficiency, Low Energy Spread, High Charge of Accelerated Electron Beams by Tailored Wakefield Plateaus for Long Driver and Witness Bunches

References

1. W.P.Leemans et al. Phys. Rev. Lett. 113 (2014) 245002.
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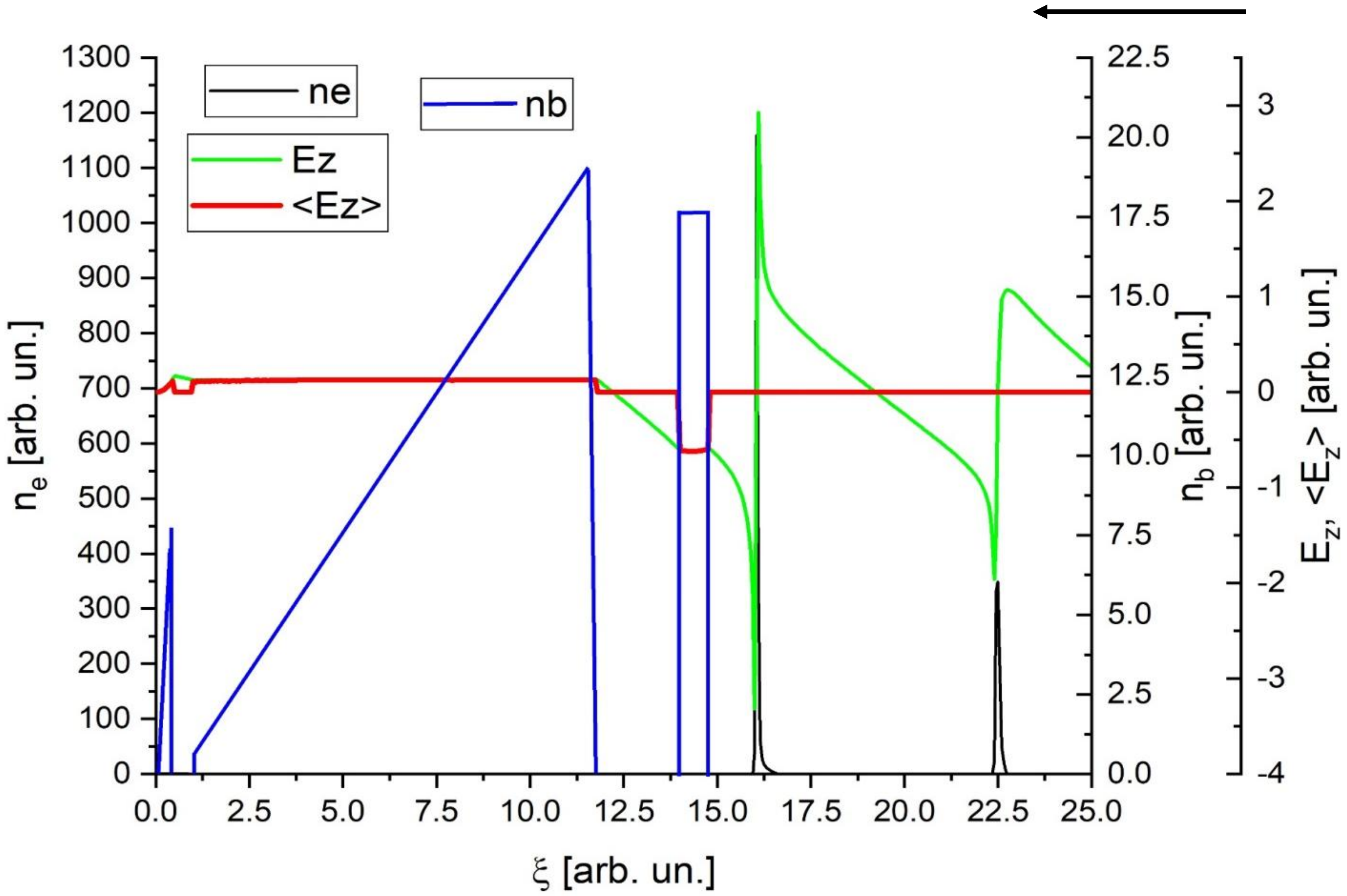
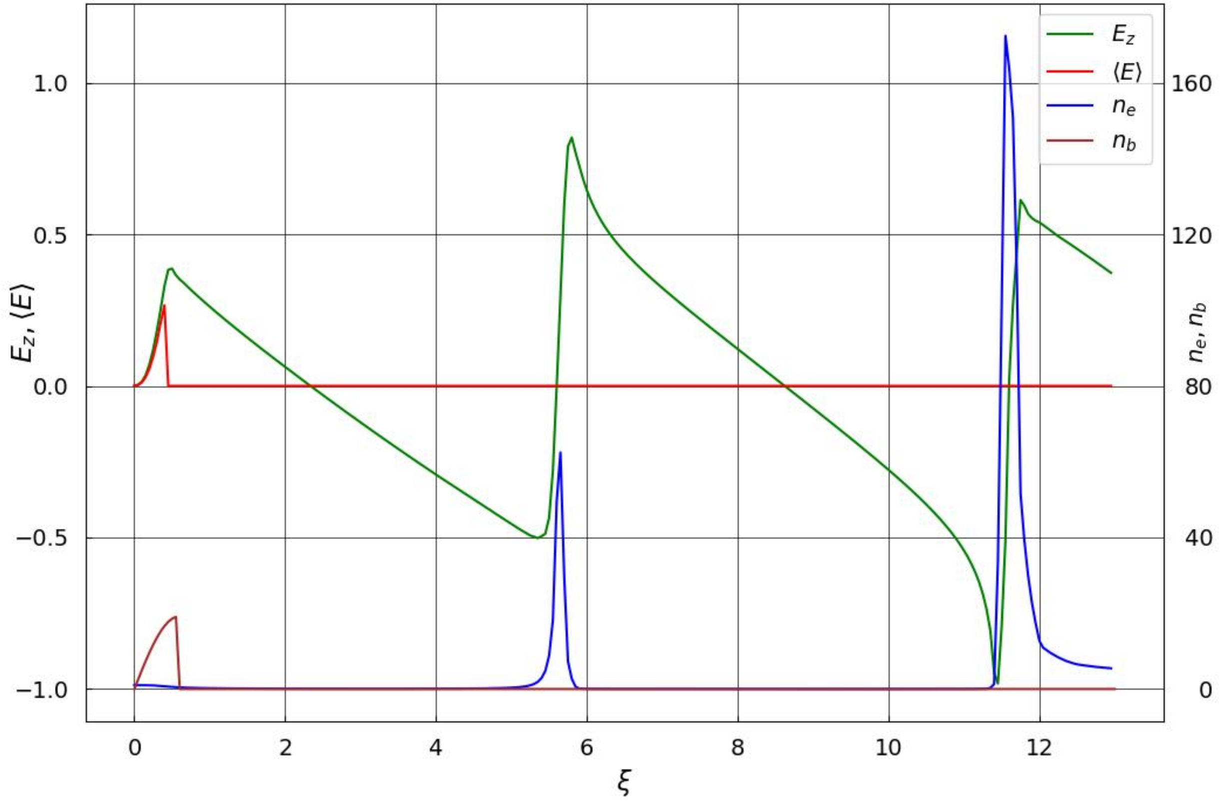
13. D.H. Whittum et al. Phys. Rev. Lett.. 67, 991 (1991).
14. T.J.Mehrling et al. Phys. Rev. Let. 118, 174801 (2017).
15. A.Martinez de la Ossa et al. Phys. Rev. Let. 121, 064803 (2018).
16. G.Loisch et al. J.Phys.: Conf. Ser. 1596, 012003 (2020)

Long Driver-bunch, **Large Transformer Ratio**, **Plateau** for Driver-bunch and Plateau for Witness-bunch but **Small Charge** of Accelerated Electrons and **Small Accelerator Efficiency**



driver's movement direction

Regularization and **Advantages** of Wakefield in a Weakly Nonlinear Regime with Narrow Bunches



from V. Maslov, R. Ovsiannikov, Numerical simulation of plateau formation by an electron bunch on the distribution of an accelerating wakefield in a plasma

Simultaneous requirements:

- the long profiled driver-bunches;
- a large charge of accelerated electrons;
- large transformer ratio;
- high efficiency.

1d $$\frac{dE_z}{dz} \approx 4\pi e n_b \;\rightarrow\; E_z \sim \xi - \xi_w(t),$$

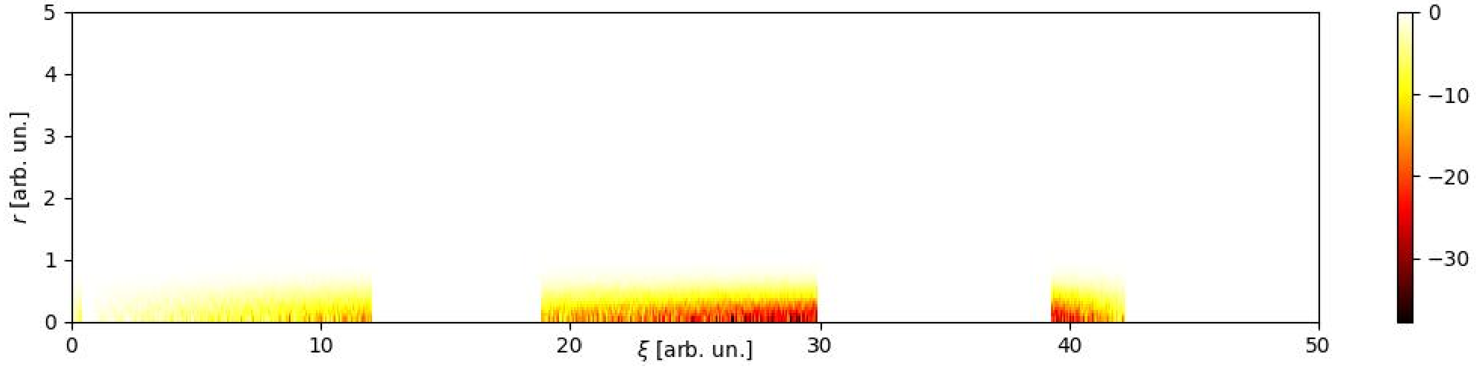
3d $$4\pi r^2 E_r = \frac{4\pi}{3} e n_i r^3$$

$$E_r = E_z = \frac{e n_i}{3} r \sim r \sim z - z_{bub}.$$

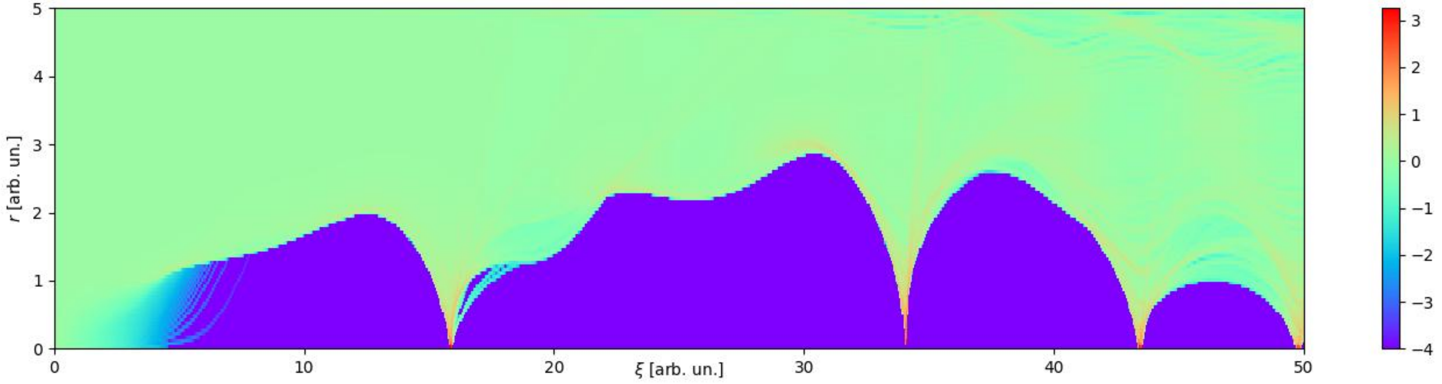
$$E_r = E_z = -\frac{e n_b}{3} r \sim -r \sim z_b - z.$$

Modified and Developed Schemes

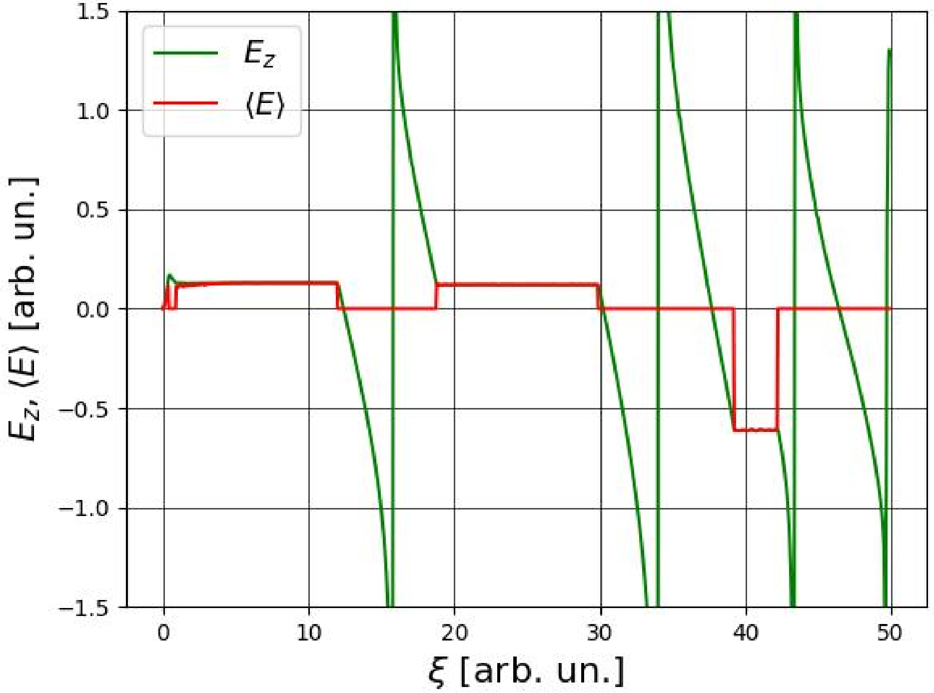
Large Charge of Accelerated Electrons at **Large Transformer Ratio** and **High Efficiency** at Plateaus in the Entire Cross-sections of Two Driver-bunches in the Decelerating Wakefield for Two Long Driver-bunches and for the Witness-bunch in the Accelerating Wakefield



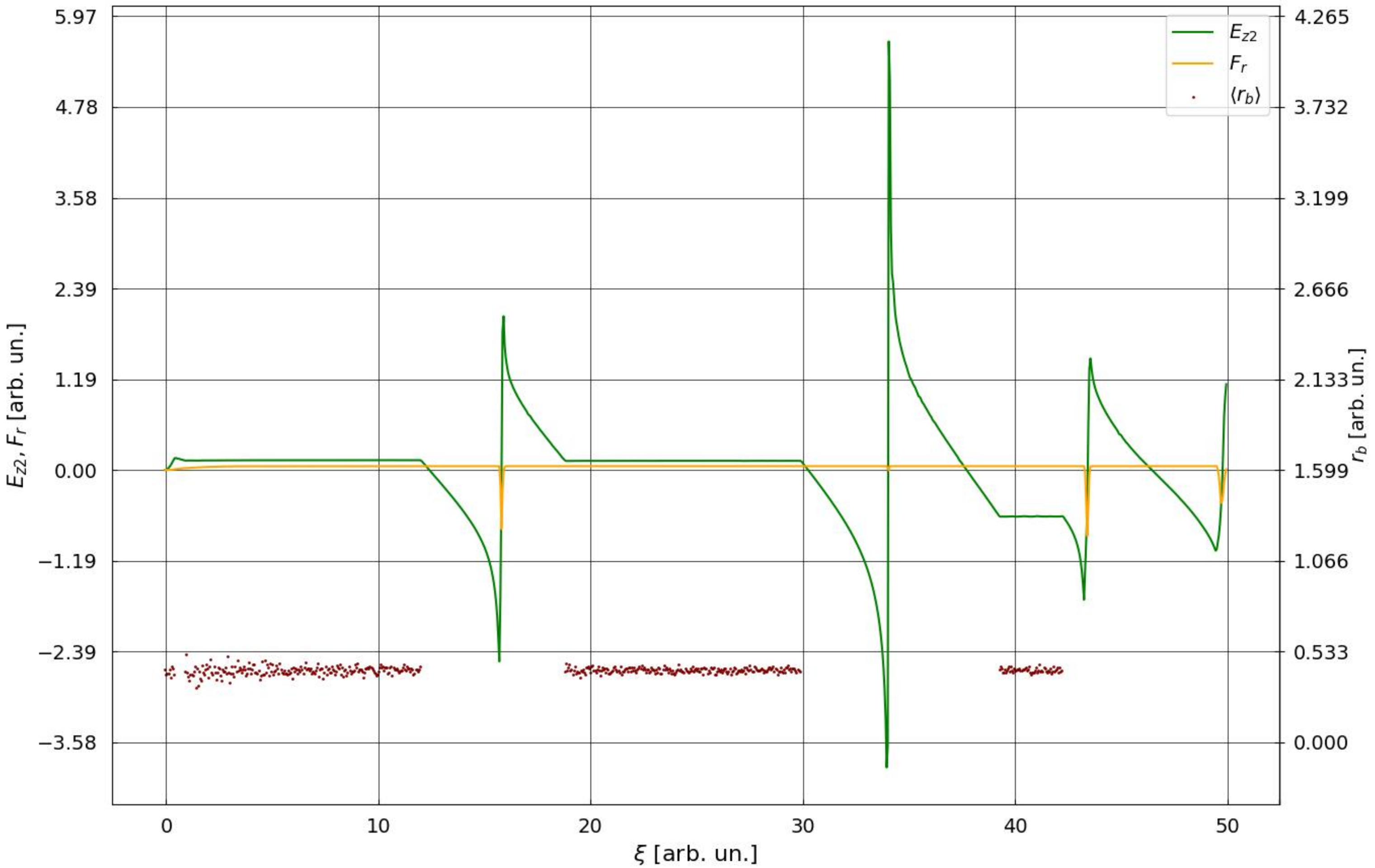
Spatial distribution of the density of two long profiled driver-bunches and of the profiled wintess-bunch



Spatial distribution of the plasma electron density, excited by two long profiled driver-bunches and reconstructed by the profiled wintess-bunch in blowout regime

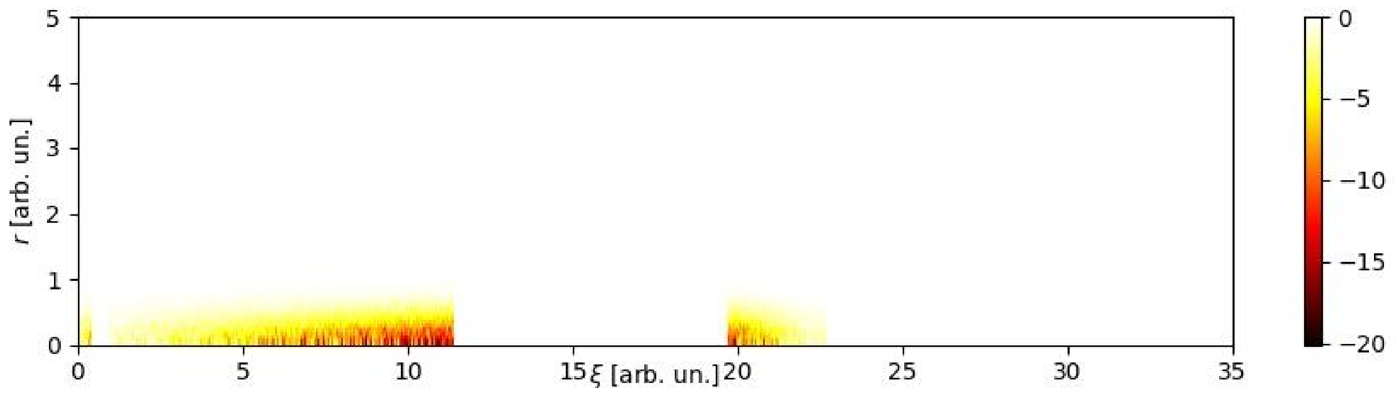


A wakefield that is independent on the longitudinal coordinate and radius along two entire long profiled driver-bunches and identical accelerating wakefield for witness-bunch. The transformer ratio is 4.5

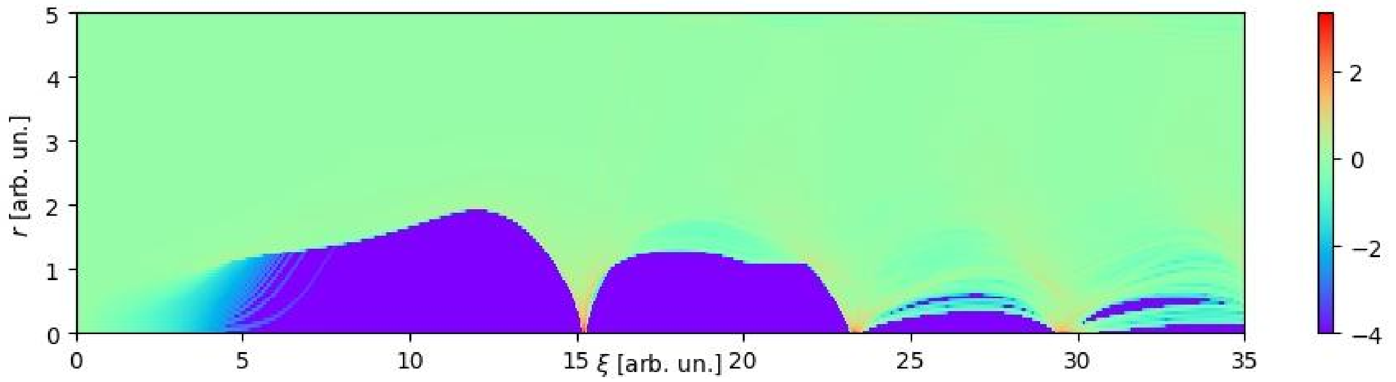


The off-axis focusing force F_r is shown by orange. The off-axis longitudinal wakefield E_z is shown by green. The brown dots show the locations of the bunches

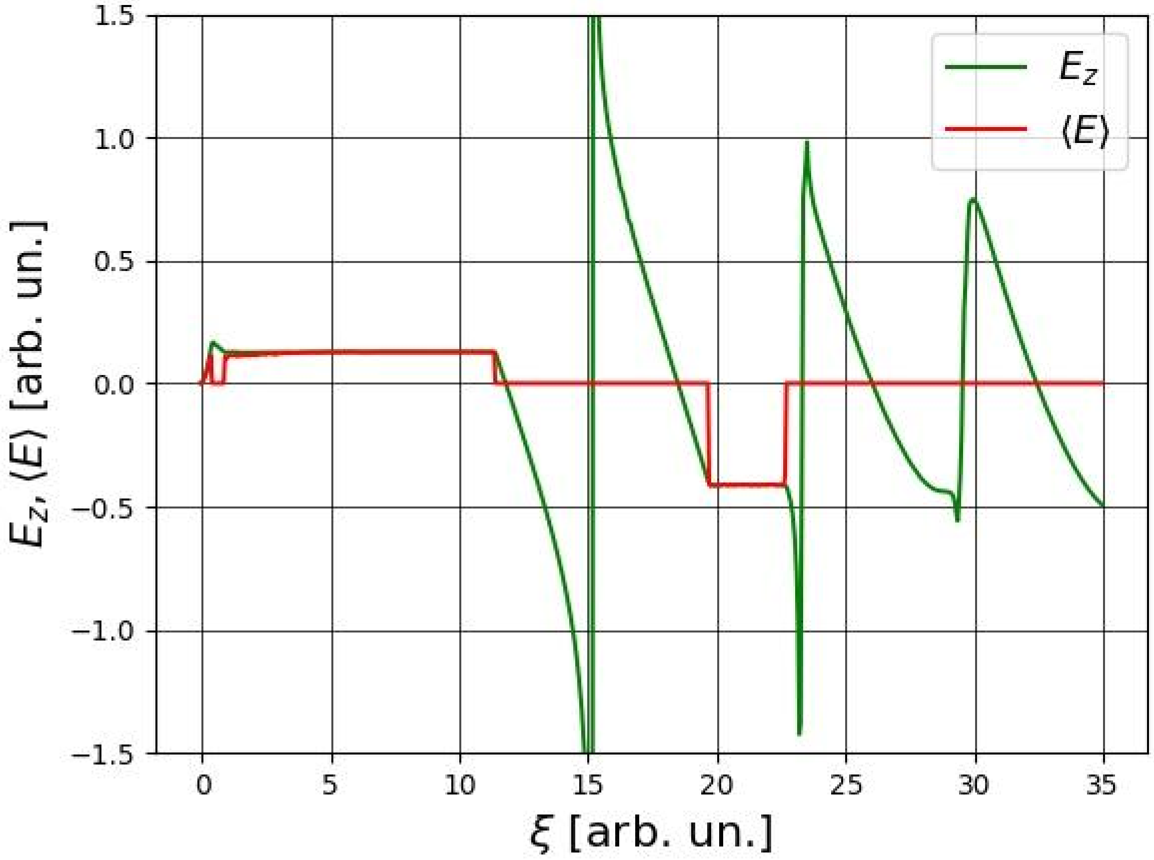
Large Charge of Accelerated Electrons at **Large Transformer Ratio** and **High Efficiency** at Plateau in the Entire Cross-section of the Driver-bunch in the Decelerating Wakefield for the Long Driver-bunch and for the Witness-bunch on the Accelerating Wakefield **for One Driver-bunch and One Witness-bunch**



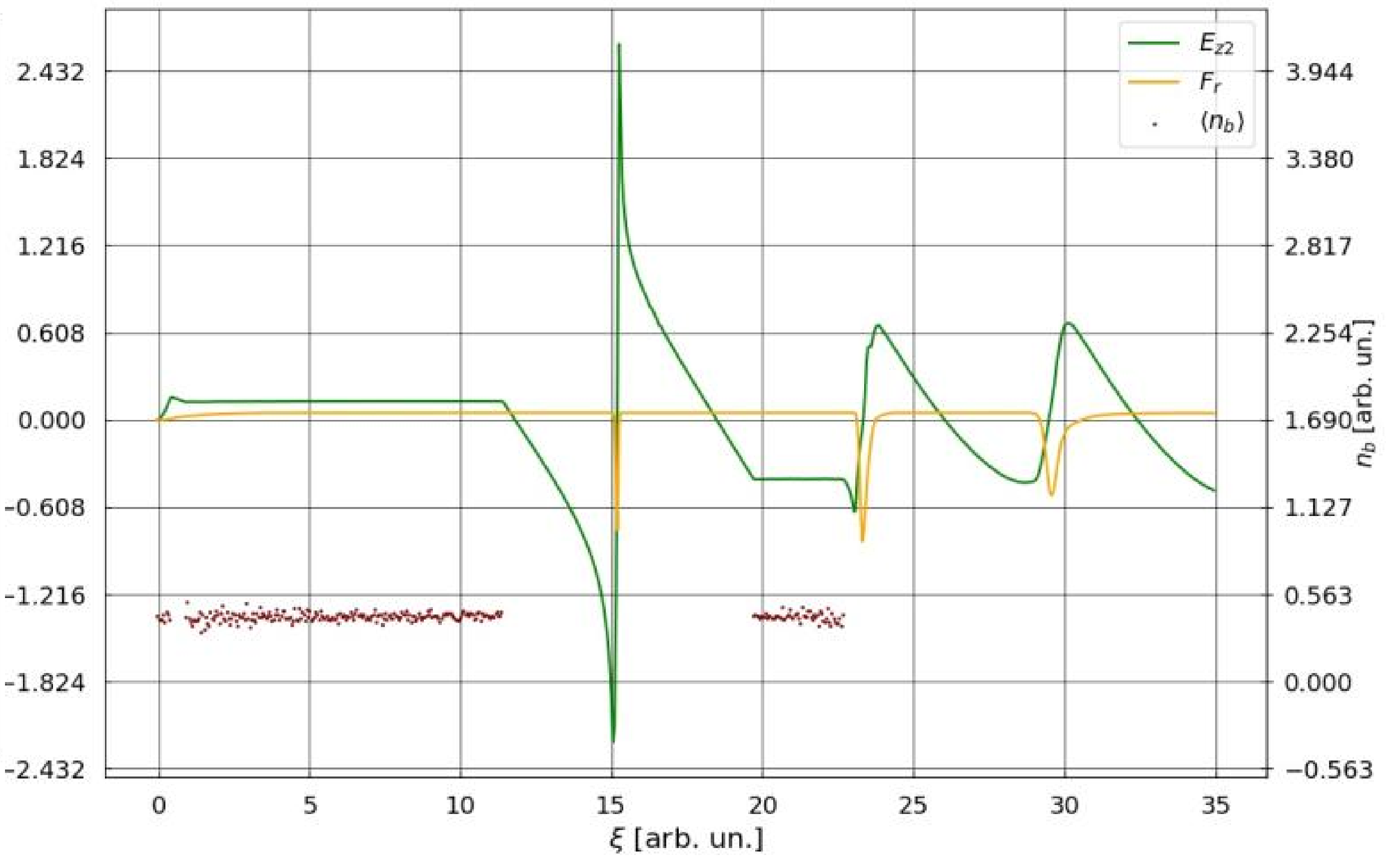
Spatial distribution of the density of two long profiled driver-bunches and of the profiled wintess-bunch



Spatial distribution of the plasma electron density, excited by two long profiled driver-bunches and reconstructed by the profiled wintess-bunch in blowout regime

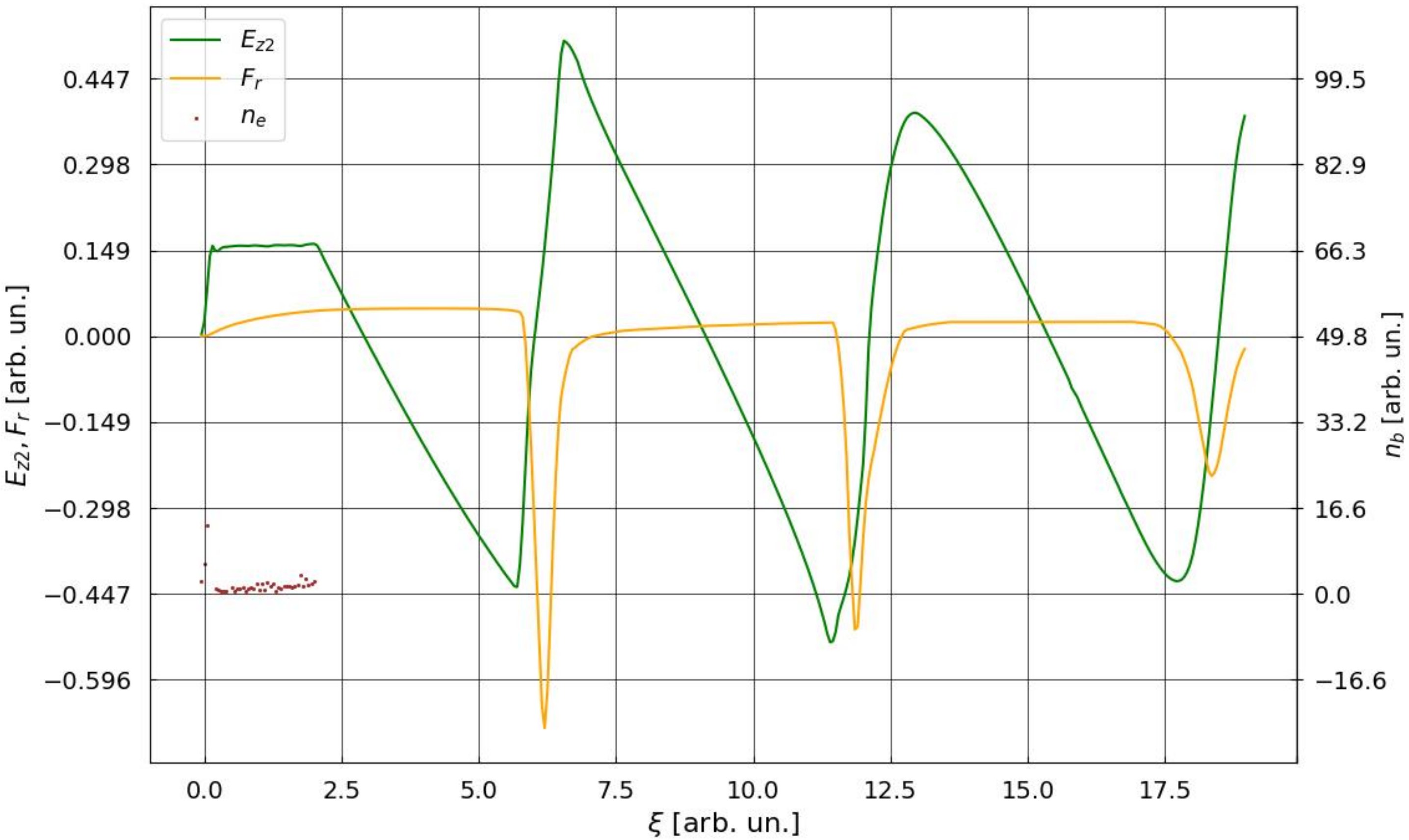


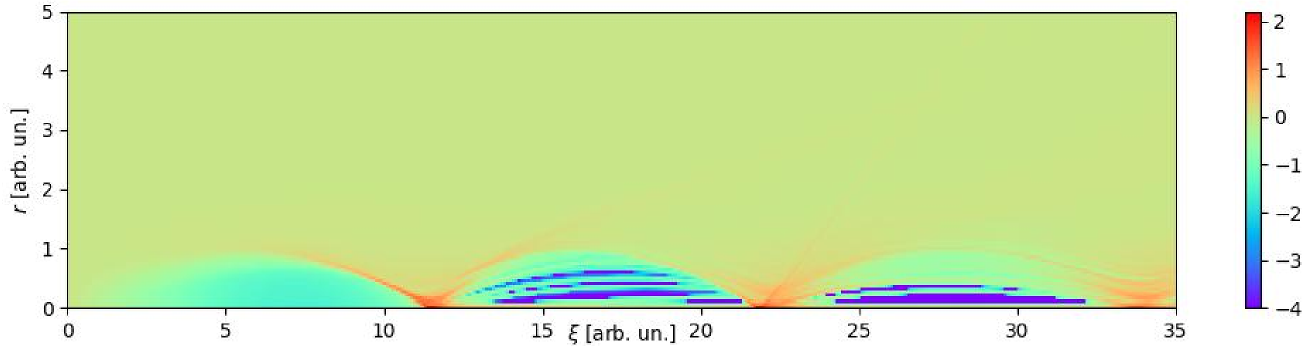
A wakefield that is **almost independent** of the longitudinal coordinate and radius along the entire long profiled driver-bunch and identical accelerating wakefield for witness-bunch



The off-axis focusing force F_r is shown by orange. The off-axis longitudinal wakefield E_z is shown by green. The brown dots show the locations of the bunches

Weakly Nonlinear Mode with Narrow Bunches





Analytical investigations

T.J.Mehrling, R.A.Fonseca, A.Martinez de la Ossa, J.Vieira. Phys. Rev. Lett. 118(2017)174801;

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Angel Ferran Pousa. Private communication

Conclusions

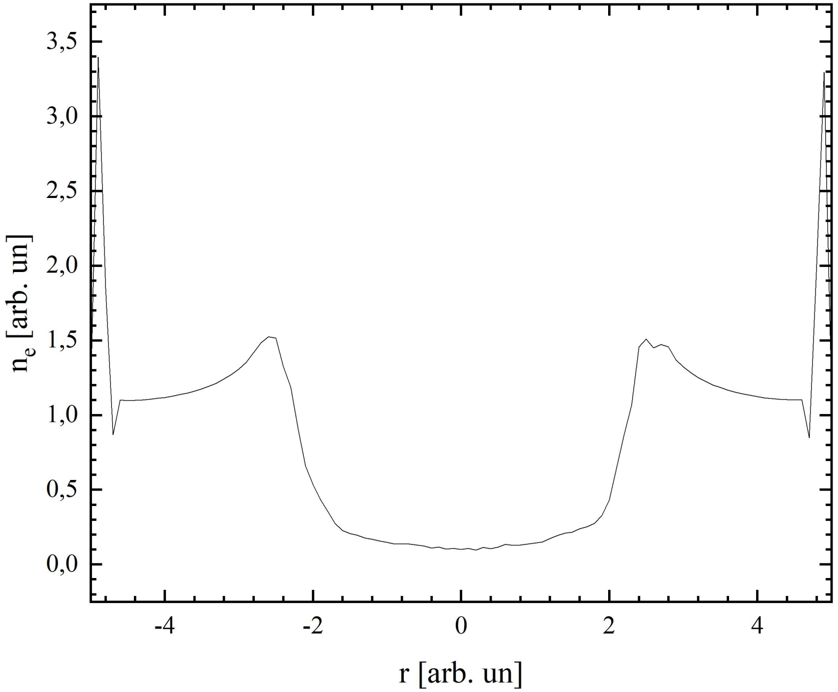
This paper discusses the results of simulations of several different driver-bunches and wintess-bunches systems that are designed to solve the following problems.

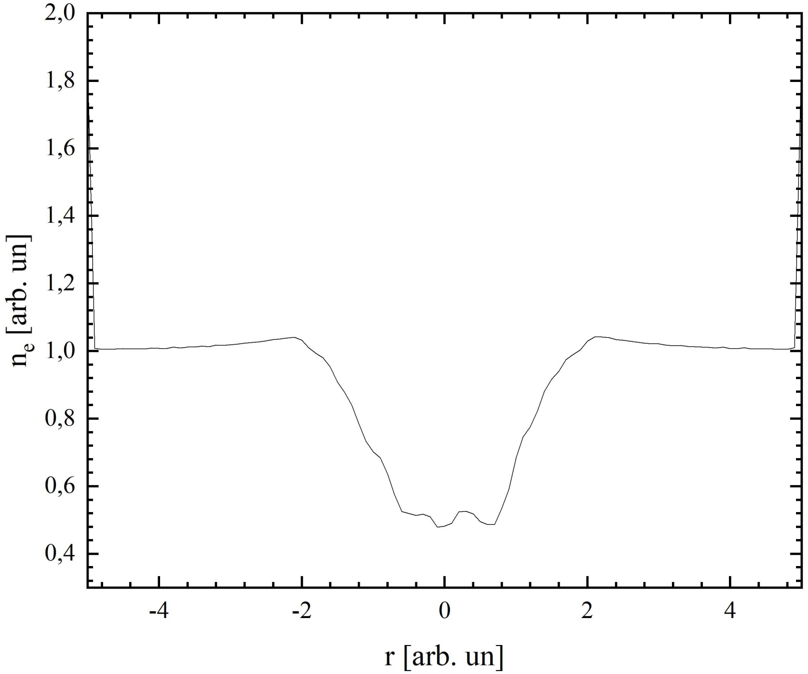
1. It is possible to obtain systems in which in the region of all narrow bunches the accelerating and decelerating electric fields have a homogeneous, plateau-like character. This makes it possible to achieve the low emittance and energy spread, high efficiency, absence of defocusing of the bunches during acceleration.

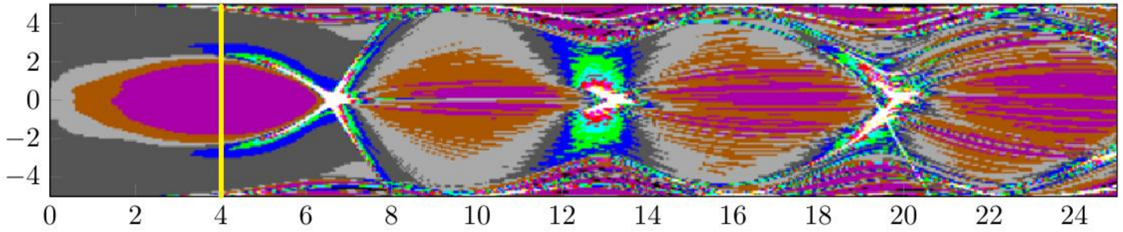
2. It is possible to obtain the driver-bunches and witness-bunches of large charge.

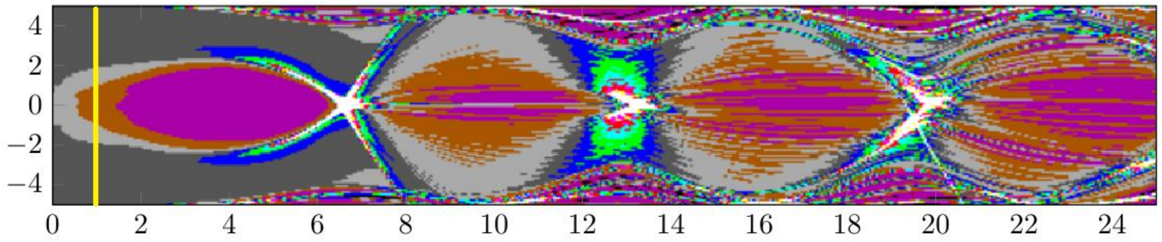
3. It is possible to achieve large transformer ratio and high efficiency.

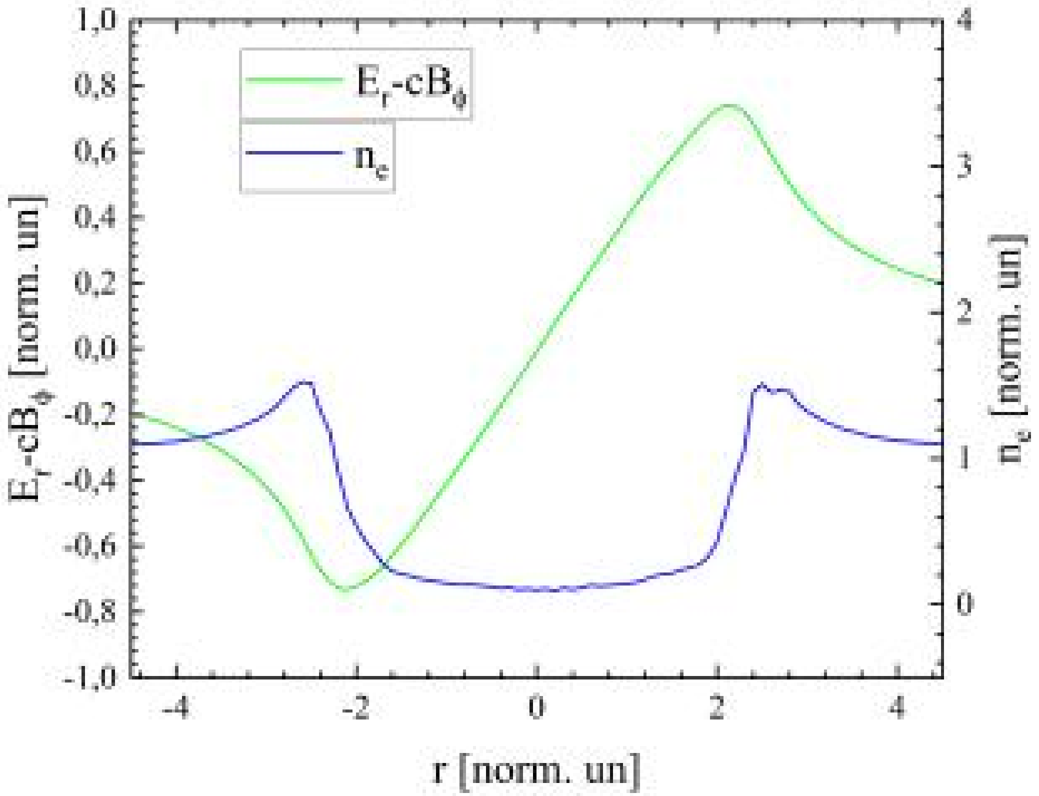
Electron distribution in the weakly nonlinear regime

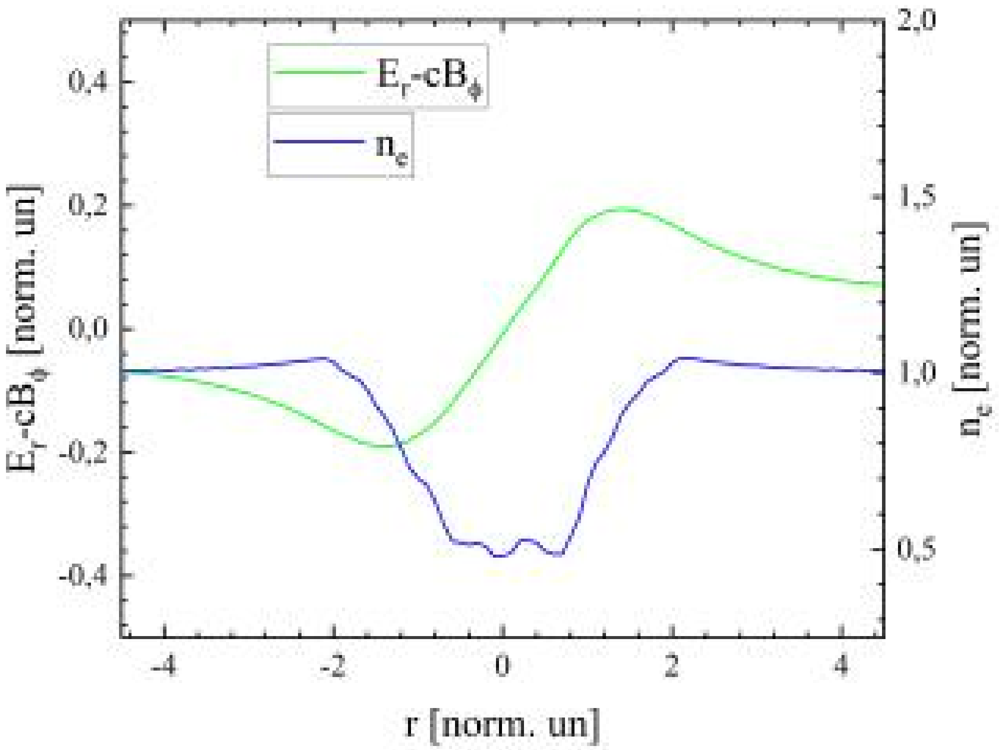









E$_r$-cB$_\phi$
n$_e$
E$_r$-cB$_\phi$ [norm. un]
n$_e$ [norm. un]
r [norm. un]


E$_r$-cB$_\phi$
n$_e$
E$_r$-cB$_\phi$ [norm. un]
n$_e$ [norm. un]
r [norm. un]

Based on the equations of motion in a cylindrical coordinate system and Maxwell's equations, we obtain the following system of equations of motion:

$$\begin{cases} \ddot{r} + \frac{A}{\gamma}\dot{r} - \dot{\phi}^2 r + r\frac{\omega_p^2}{2\gamma}\left[\eta_i - \eta_e - \frac{1}{c}\beta_{z,p}\right] = 0, \\ r\ddot{\phi} + 2\dot{r}\dot{\phi} + \frac{A}{\gamma}r\dot{\phi} = 0, \\ \ddot{z} + \frac{A}{\gamma}\dot{z} - \frac{eA}{\gamma} = 0, \end{cases}$$

where $A = -\frac{eE_z}{mc} = \dot{\gamma}$, $\eta_{i,e}$ is the normalized to the unperturbed density n_0 density of ions and electrons, $\beta_{z,p}$ is the normalized plasma current, ω_p is the plasma frequency.

Estimation of the radial oscillation period. The influence of anharmonicity

Виконаємо наступне перетворення координат та часу:

$$\begin{cases} x \to x' = \alpha x \\ t \to t' = \beta t \end{cases}$$

У випадку гармонічних коливань:

$$L = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2 \to L' = \frac{\alpha^2}{\beta^2}\frac{m}{2}\dot{x}^2 - \alpha^2\frac{k}{2}x^2.$$

При $\frac{\alpha^2}{\beta^2} = \alpha^2$, коли $\beta = 1$ маємо еквівалентні функції Лагранжа.

У випадку ангармонічних коливань:

$$L = \frac{m}{2}\dot{x}^2 - \frac{k}{2}x^2 + ax^3 + bx^4 + \ldots.$$

У цьому випадку зрозуміло, що не існує жодних нетривіальних перетворень координати та часу, для яких можна було б отримати еквівалентні функції Лагранжа.

Estimation of the radial oscillation period

If we set $\dot{\phi} = 0$, we obtain the following equation of motion:

$$\ddot{r} + \frac{A}{\gamma}\dot{r} + r\frac{\omega_p^2}{2\gamma}\left[\eta_i - \eta_e - \frac{1}{c}\beta_{z,p}\right] = 0.$$

For the evaluation, we assume that $\gamma = Const$:

$$\ddot{r} + r\frac{\omega_p^2}{2\gamma}\left[\eta_i - \eta_e - \frac{1}{c}\beta_{z,p}\right] = 0.$$

This equation corresponds to the following integral of motion:

$$\mathcal{E} = \dot{r}^2 + \int drr\frac{\omega_p^2}{\gamma}\left[\eta_i - \eta_e - \frac{1}{c}\beta_z\right].$$

Estimation of the radial oscillation period

We get the following period:

$$T = 4\int_0^{r_0} \frac{dr}{\sqrt{\mathcal{E} - \int drr\frac{\omega_p^2}{\gamma}\left[\eta_i - \eta_e - \frac{1}{c}\beta_z\right]}}.$$

The ion density is uniform, therefore:

$$T = 4\int_0^{r_0} \frac{dr}{\sqrt{\mathcal{E} - \int drr\frac{\omega_p^2}{\gamma}[1 - \eta_e(r)]}}.$$

The final formula looks like this:

$$T = 4\frac{\sqrt{2\gamma}}{\omega_p}\int_0^1 \frac{dx}{\sqrt{1 - x^2 + 2\left[\int_0^x dt\, t\, \eta_e(t) - \int_0^1 dx\, x\, \eta_e(x)\right]}}.$$

Estimation of the radial oscillation period. Model distribution

Based on numerical simulation, we can propose the following model electron distribution:

$$\eta_e(x) = a + b\sin^2 kr_0 x\,,$$

here k is the radial wavenumber.

Let us consider the asymptotics for small r:

$$T = 8\frac{\sqrt{\gamma}}{\omega_p\sqrt{2-a-b\alpha}}F\left(\frac{\pi}{2},\sqrt{\frac{b\alpha}{2-a-b\alpha}}\right),$$

here F is the elliptic integral of the 1st kind.

Estimation of the radial oscillation period. Model distribution

If we consider the case that all the plasma electrons have left the axis, we obtain the following expression:

$$T = 8\frac{\sqrt{\gamma}}{\omega_p\sqrt{2-\alpha b}}F\left(\frac{\pi}{2},\sqrt{\frac{\alpha b}{2-\alpha b}}\right).$$

In the zero approximation:

$$T = 4\frac{\sqrt{2\gamma}}{\omega_p}F\left(\frac{\pi}{2},0\right) = \frac{2\pi\sqrt{2\gamma}}{\omega_p} \Longrightarrow \omega = \frac{\omega_p}{\sqrt{2\gamma}} = T_0.$$

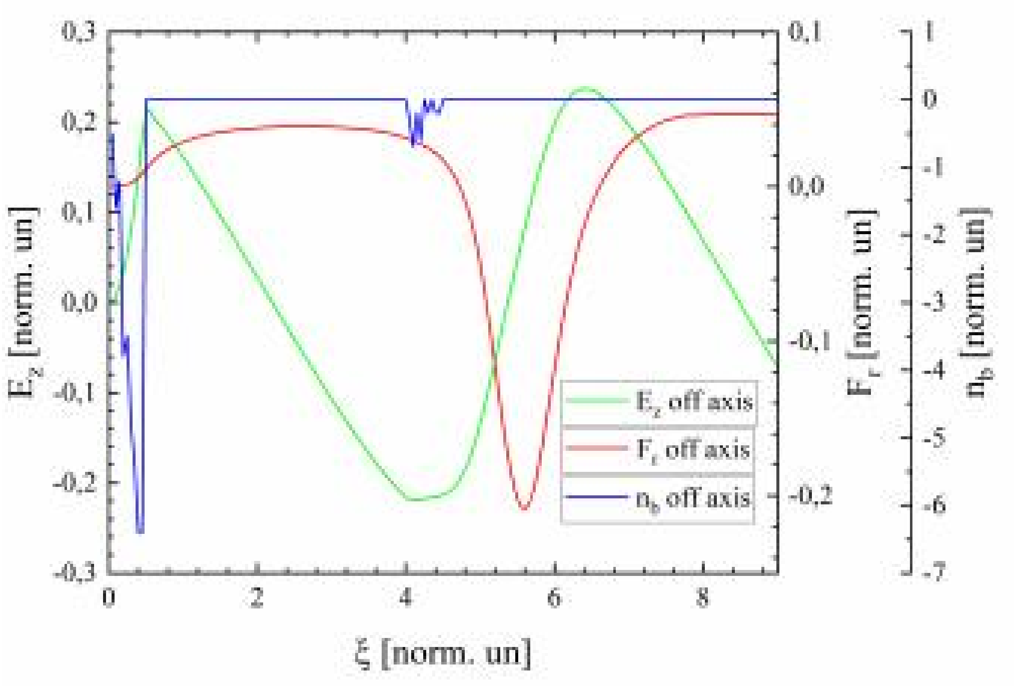
Estimation of the radial oscillation period. Model distribution

Next member of the series:

$$T = 8\frac{\sqrt{\gamma}}{\omega_p\sqrt{2-bk^2r_0^2}}\frac{\pi}{2}\left(1+\frac{1}{4}\frac{bk^2r_0^2}{2-bk^2r_0^2}\right) \approx 2\pi\frac{\sqrt{2\gamma}}{\omega_p}\left(1+\frac{1}{8}bk^2r_0^2\right).$$

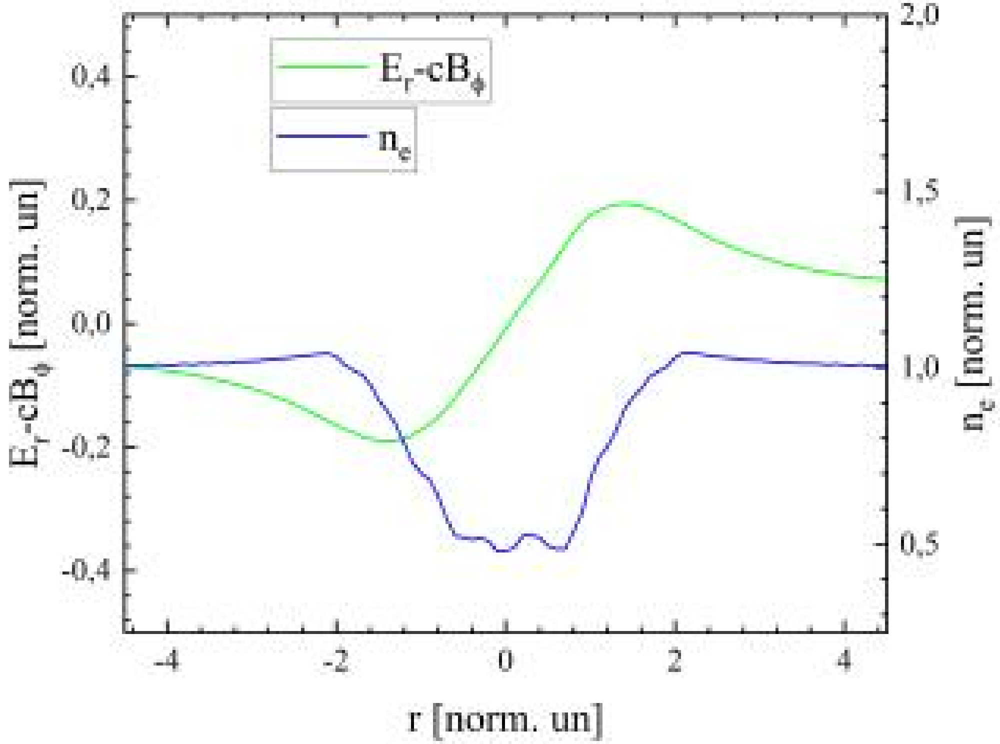
So, as we see, the period increases in proportion to the square of the amplitude, which is to be expected.
If the amplitude of the electron oscillations is so large that the electrons approach the separatrix, then in this case the period of oscillations will approach infinity, since when passing through the separatrix the electron will enter a state of infinite motion.

Condition of stochasticity



$$2\omega_r^{-1} R \partial_r \omega_r \geq 1$$

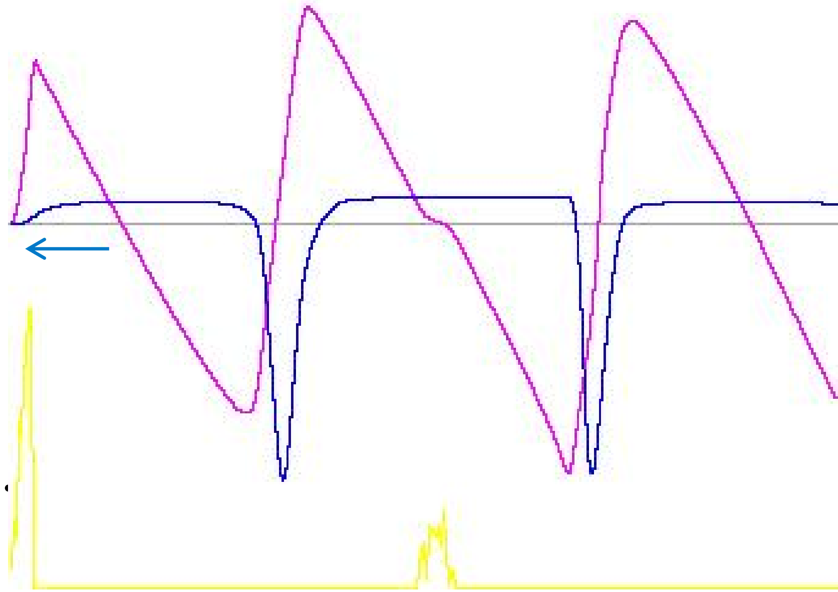
$$\frac{2\sqrt{2\gamma}}{\omega_p} \delta\omega \geq 1$$



Conclusions

Thus, it is shown that, taking into account the non-uniform distribution of plasma electrons that have not yet left the wake bubble, the electronic oscillations become desynchronized. Namely, at that distance from the driver head, where all plasma electrons have not yet left the axis, the driver bunch can be continuous, remaining stationary due to phase mixing during asynchronous radial oscillations. At that distance from the driver head, where all plasma electrons have already left the axis, but not all plasma electrons have left the bubble, the driver bunch can be stationary due to phase mixing during asynchronous radial oscillations, if it is hollow. Thus, in order for the bunch to be stationary due to phase mixing during asynchronous radial oscillations, it must be in the form of a hollow cone with a finite wall thickness. In this case, the side of the cone should be directed approximately along the edge of the wake bubble.

Plasma Lens, which Reduces the Energy Spread of Gaussian - Kind Bunch, in Linear and Nonlinear Regime



Focusing of accelerated electron and positron bunches in plasma
is very important (see

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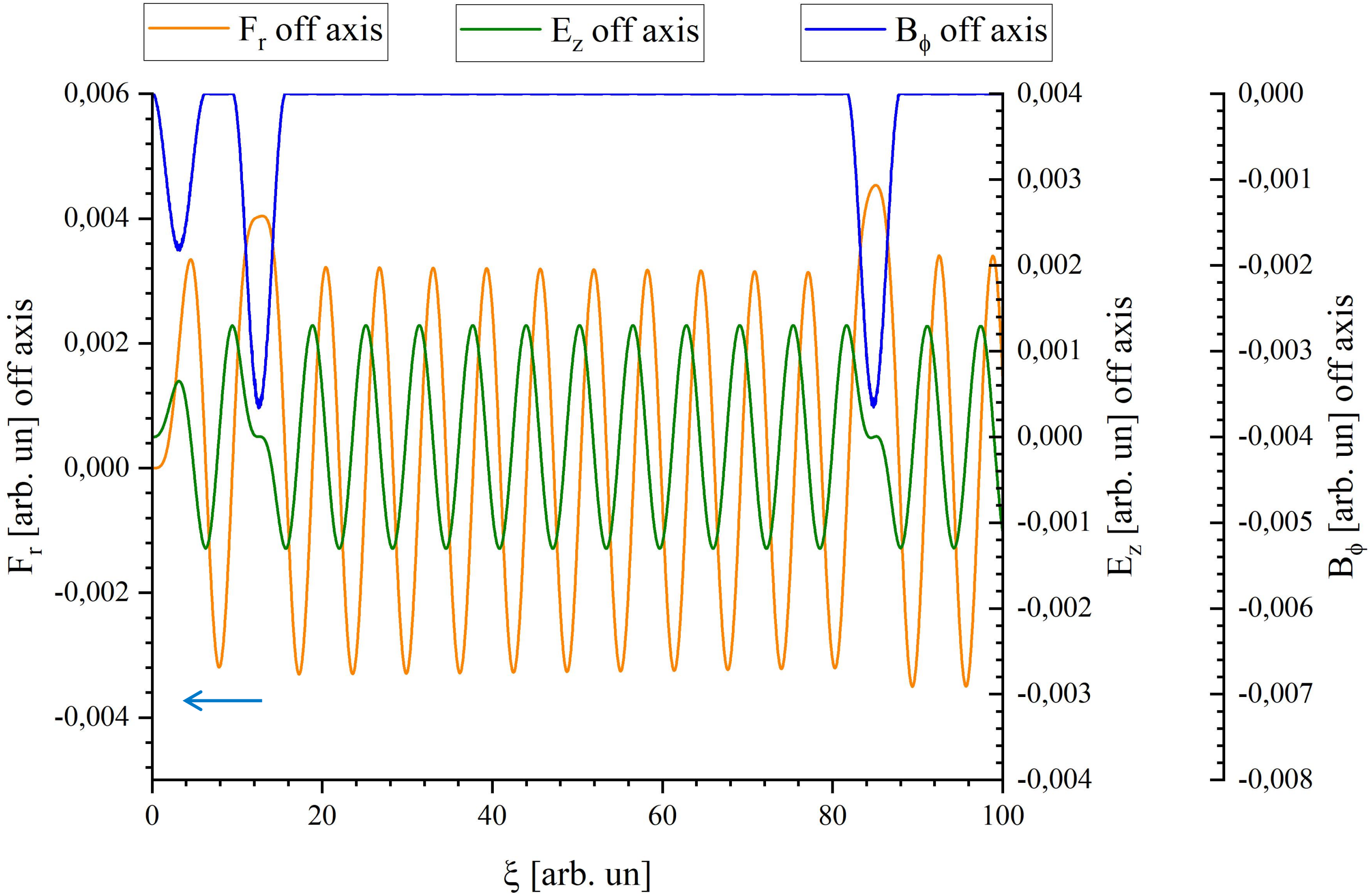
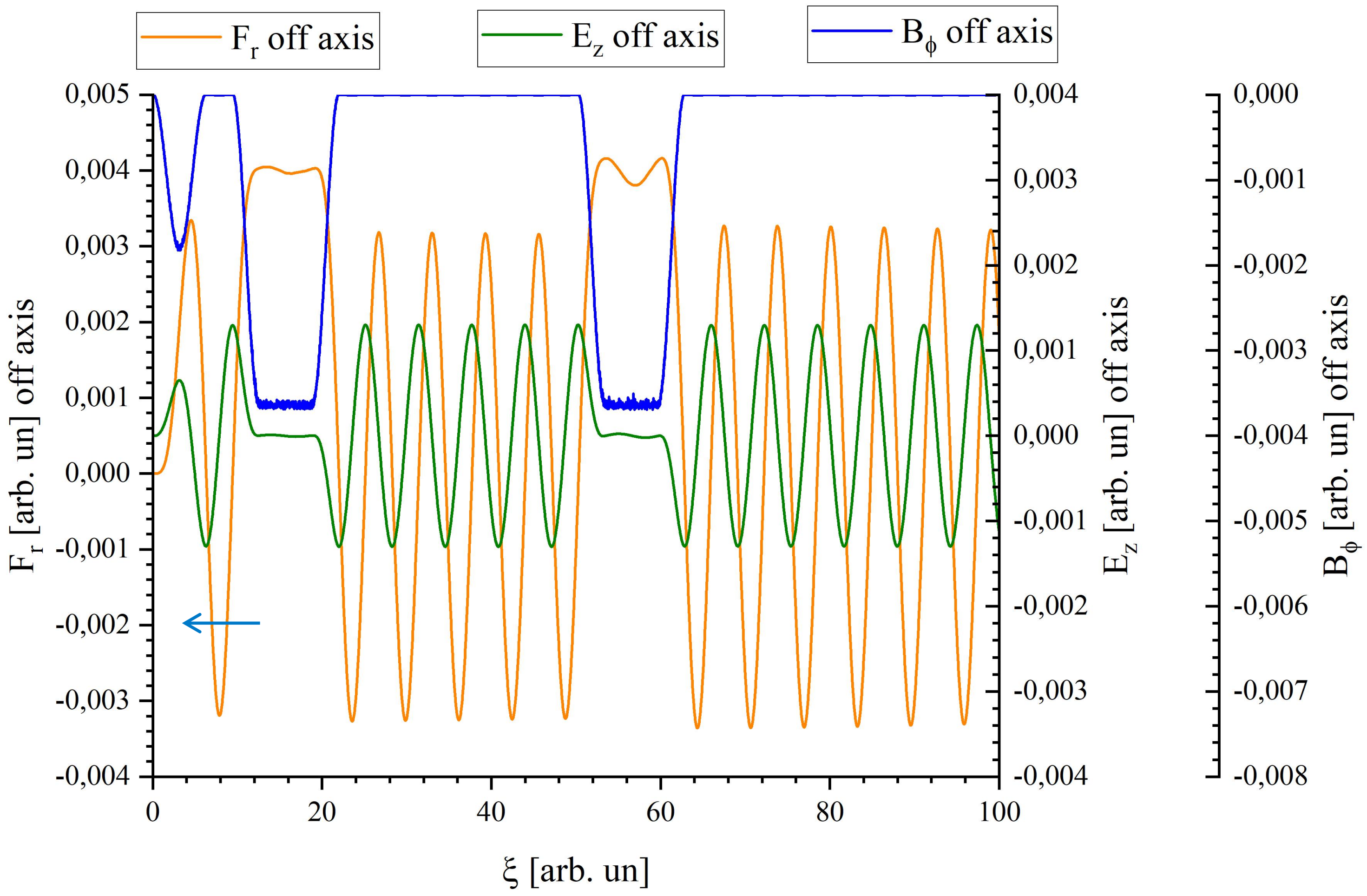
9. V.Maslov et al. EEJP. 2 (2019) 64.
10. G.Hairapetian et al. Phys. Plasma. 2 (1995) 2555.

Plasma lens for a Gaussian-like bunch (1 wavelength)



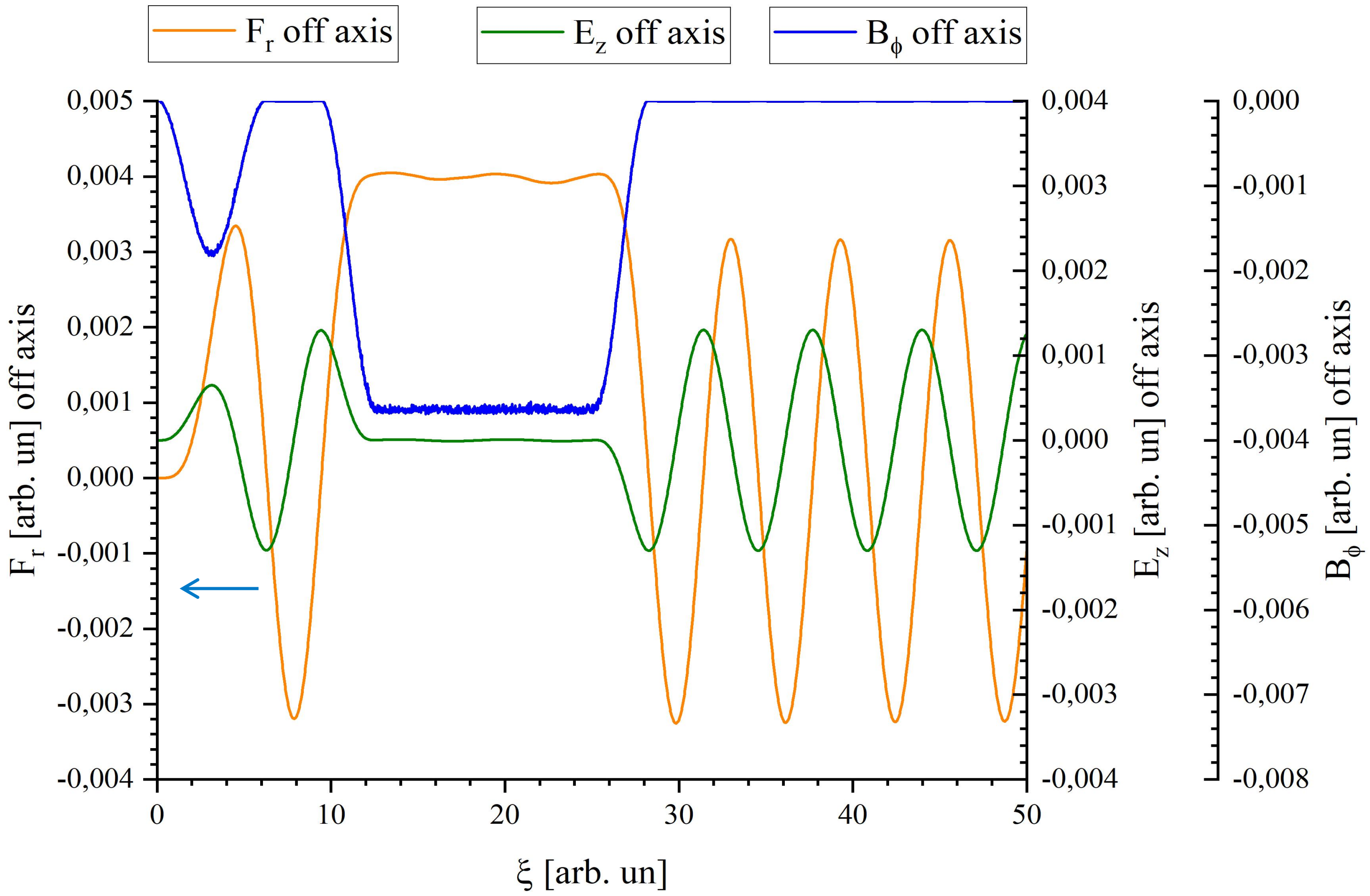
$E_z(\xi)$, $F_r(\xi)$, $B_\phi(\xi)$, $\xi = ct - z$ for a Gaussian-like precursor-bunch (length $l_{pr} = \lambda$ *(on the basis)* and density $n_{pr} = n_b/2$, n_b is the density of the main bunches) and two main distant Gaussian-like bunches, the length of which is $l_b = \lambda$ *(on the basis)*, moving to the left. The density in the transverse direction has a Gaussian distribution

Plasma lens for an inhomogeneous bunch (2 wavelengths)



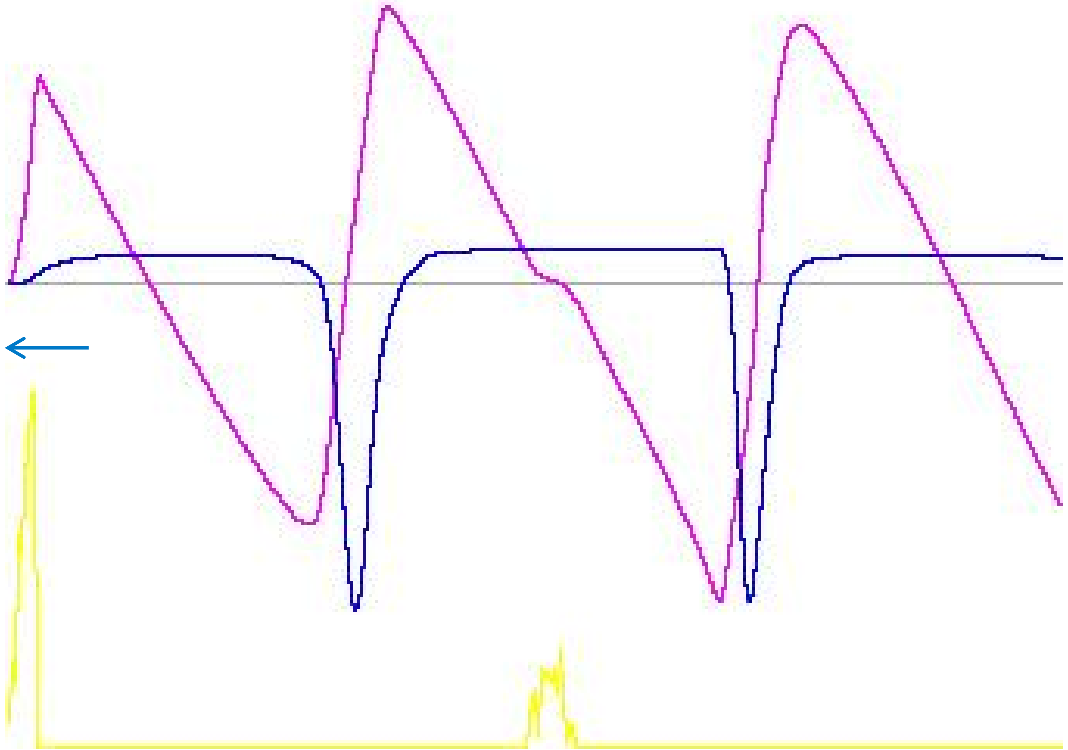
$E_z(\xi)$, $F_r(\xi)$, $B_\phi(\xi)$, $\xi = ct - z$ for a Gaussian-like precursor-bunch (length $l_{pr} = \lambda$ and density $n_{pr} = n_b/2$, n_b is the density of the main bunch) and two main distant Gaussian-like bunches, the length of which is $l_b = 2\lambda$, moving to the left. Each Gaussian-like front of the main bunches has a length of $\lambda/2$. The main part is uniform and has a length of λ.

Plasma lens for an inhomogeneous bunch (3 wavelengths)

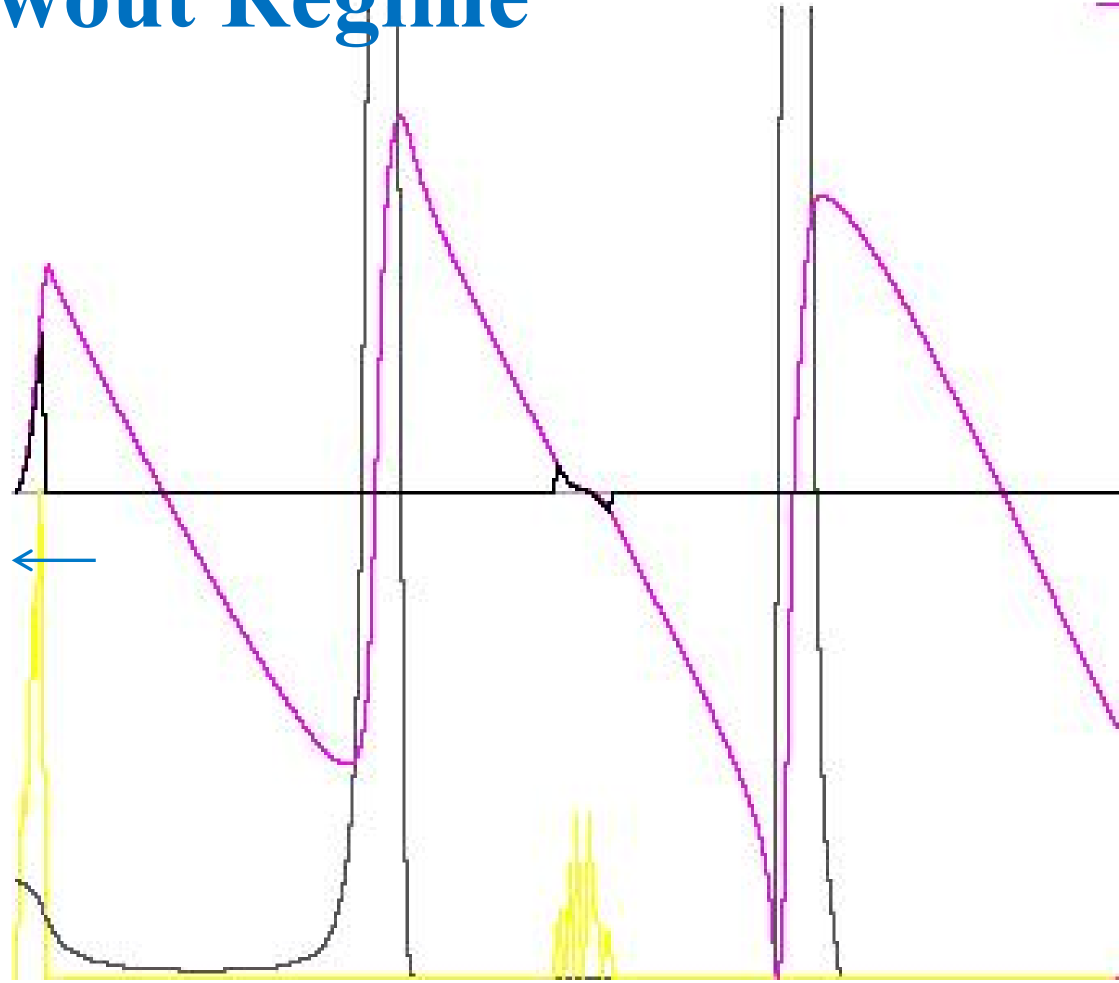


$E_z(\xi)$, $F_r(\xi)$, $B_\phi(\xi)$, $\xi = ct - z$ for a Gaussian-like precursor-bunch (length $l_{pr} = \lambda$ and density $n_{pr} = n_b/2$, n_b is the density of the main bunch) and main Gaussian-like bunch, the length of which is $l_b = 3\lambda$, moving to the left. Each Gaussian-like front of the main bunches has a length of $\lambda/2$. The main part is uniform and has a length of 2λ.

Plasma Lens, which Reduces the Energy Spread of Gaussian - Kind Bunch, in Blowout Regime



$E_z(\xi, r = r_b)$, $F_r(\xi)$, $n_b(\xi)$ for a Gaussian-like precursor-bunch and main Gaussian-like bunch, moving to the left



$E_z(\xi, r = 0)$, $< E_z(\xi) >$, $n_e(\xi)$, $n_b(\xi)$ for a Gaussian-like precursor-bunch and main Gaussian-like bunch, moving to the left

Conclusions

Therefore, we have demonstrated that the precursor-bunch is necessary for more uniform focusing and that the witness does not lose energy by creating a plasma lens. The plasma lens reduces the energy spread of the bunch, as the head of the bunch, which has more energy, loses energy and its tail, which has less energy, accelerates. We have shown that in the linear regime it is possible to focus bunch with a length longer than the wavelength.

Thank you

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